

# 5.7 The Second Derivative Test

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 9 marks

## Objective

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Build the skills to answer exam questions on **the Second Derivative Test**.

**You must be able to:**

- use  $f''$  at a critical point to classify a **local extremum** 局部极值

## 1 Worked examples

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Study these first. Each one shows the method for a question type used later.

### ■ Second Derivative Test

At a critical point  $c$  where  $f'(c) = 0$ :

- $f''(c) > 0 \rightarrow$  local **minimum** (concave up);
- $f''(c) < 0 \rightarrow$  local **maximum** (concave down);
- $f''(c) = 0 \rightarrow$  **inconclusive**.

### ■ Example

$f(x) = x^2 - 4x$ :  $f''(x) = 2 > 0$ , so the critical point is a local minimum.

## 2 Practice

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**2.1** State the Second Derivative Test for a local minimum. [1]

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**2.2** For  $f(x) = x^2 - 4x$  with  $f'' = 2$ , classify the critical point. [2]

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**2.3** State when the test is inconclusive. [1]

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### 3 Exam-style questions

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3.1 If  $f'(c) = 0$  and  $f''(c) < 0$ , then  $c$  is a [1]

- **A** local minimum
  - **B** local maximum
  - **C** inflection point
  - **D** nothing
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3.2 If  $f'(c) = 0$  and  $f''(c) > 0$ , then  $c$  is a [1]

- **A** local maximum
  - **B** local minimum
  - **C** endpoint
  - **D** asymptote
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3.3  $f(x) = x^2 - 6x$ , with  $f'(3) = 0$ .

(a) Find  $f''(x)$ . [1]

(b) Evaluate  $f''(3)$ . [1]

(c) Classify  $x = 3$ . [1]

### 4 Go further

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- work through the **5.7 Using the Second Derivative Test to Determine Extrema** lesson on the **Learn** page;
- read the **Analytical Applications of Differentiation** section of the AP Calculus AB handout on the **Know** page.

## Solutions

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**2.1** if  $f'(c) = 0$  and  $f''(c) > 0$ , then  $c$  is a local minimum.

**2.2**  $f'' = 2 > 0$ , so it is a local minimum.

**2.3** when  $f''(c) = 0$ .

**3.1 B.**

**3.2 B.**

**3.3** (a) 2. (b)  $2 > 0$ . (c) local minimum.