AP CALCULUS AB • EXERCISE SHEET

5.6 Concavity and Points of Inflection

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **determining concavity**.

You must be able to:

- use the sign of f'' to find **concavity** 凹凸性
- identify an **inflection point** 拐点

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Concavity**

- f is **concave up** where $f'' > 0$;
- f is **concave down** where $f'' < 0$.

An **inflection point** is where concavity changes (and f'' changes sign).

■ **Example**

$f(x) = x^3$: $f''(x) = 6x$, which is > 0 for $x > 0$; the inflection point is at $x = 0$.

2 Practice

2.1 State where f is concave up. [1]

2.2 For $f(x) = x^3$, find f'' and where it is concave up. [2]

2.3 State what an inflection point is. [1]

3 Exam-style questions

3.1 f is concave up where [1]

- **A** $f' > 0$
- **B** $f'' > 0$
- **C** $f'' < 0$
- **D** $f = 0$

3.2 An inflection point is where [1]

- **A** $f' = 0$
- **B** concavity changes
- **C** $f = 0$
- **D** f'' is constant

3.3 $f(x) = x^3$.

(a) Find $f''(x)$. [1]

(b) State where $f'' > 0$. [1]

(c) State the inflection point. [1]

4 Go further

- work through the **5.6 Determining Concavity of Functions over Their Domains** lesson on the **Learn** page;
- read the **Analytical Applications of Differentiation** section of the AP Calculus AB handout on the **Know** page.

Solutions

2.1 where $f'' > 0$.

2.2 $f''(x) = 6x$; concave up for $x > 0$.

2.3 a point where concavity changes (from up to down or vice versa).

3.1 B.

3.2 B.

3.3 (a) $6x$. (b) $x > 0$. (c) $x = 0$.