

4.7 L'Hospital's Rule for Indeterminate Forms

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS l'hospital's rule 洛必达法则 • indeterminate form 未定式

Objective

Build the skills to answer exam questions on **L'Hospital's Rule for indeterminate forms**.

You must be able to:

- recognise an **indeterminate form** 未定式 $\frac{0}{0}$ or $\frac{\infty}{\infty}$
- apply **L'Hospital's Rule** 洛必达法则

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ L'Hospital's Rule

For a limit of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$,

$$\lim \frac{f}{g} = \lim \frac{f'}{g'}$$

(when the right-hand limit exists).

■ Example

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\cos x}{1} = 1.$$

2 Practice

2.1 State the forms L'Hospital's Rule applies to. [1]

2.2 Apply it to $\lim_{x \rightarrow 0} \frac{\sin x}{x}$. [2]

2.3 State what you do to the numerator and denominator. [1]

3 Exam-style questions

3.1 L'Hospital's Rule applies to [1]

- **A** any limit
- **B** $\frac{0}{0}$ or $\frac{\infty}{\infty}$
- **C** $\frac{1}{0}$
- **D** $\frac{\frac{1}{2}}{2}$

3.2 L'Hospital's Rule replaces $\frac{f}{g}$ with [1]

- **A** $f \cdot g$
- **B** $\frac{f'}{g'}$
- **C** $\frac{f'}{g}$
- **D** $\frac{f}{g'}$

3.3 $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$.

(a) Check the form. [1]

(b) Differentiate top and bottom. [1]

(c) Evaluate. [1]

Solutions

2.1 $\frac{0}{0}$ and $\frac{\infty}{\infty}$.

2.2 $\lim_{x \rightarrow 0} \frac{\cos x}{1} = 1$.

2.3 differentiate each separately, then take the limit of the new quotient.

3.1 B.

3.2 B.

3.3 (a) $\frac{0}{0}$. (b) $\frac{e^x}{1}$. (c) 1.