

4.6 Local Linearity and Linearization

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS linearization 线性化 • local linearity 局部线性

Objective

Build the skills to answer exam questions on **local linearity and linearization**.

You must be able to:

- write the **linearization** 线性化 $L(x) = f(a) + f'(a)(x - a)$
- use it to approximate a value

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Linearization

Near $x = a$,

$$f(x) \approx L(x) = f(a) + f'(a)(x - a)$$

— the tangent line at a .

■ Example

$f(x) = \sqrt{x}$ at $a = 4$: $f(4) = 2$, $f'(4) = \frac{1}{4}$, so $L(x) = 2 + \frac{1}{4}(x - 4)$ and $\sqrt{4.1} \approx 2.025$.

2 Practice

2.1 Write the linearization formula. [1]

2.2 For $f(x) = \sqrt{x}$ at $a = 4$, state $f(4)$ and $f'(4)$. [2]

2.3 State what $L(x)$ approximates. [1]

3 Exam-style questions

3.1 The linearization of f at a is [1]

- **A** $f(a)$
- **B** $f(a) + f'(a)(x - a)$
- **C** $f'(a)(x - a)$
- **D** $f(x) \cdot a$

3.2 Linearization uses the _____ line. [1]

- **A** secant
- **B** tangent
- **C** normal
- **D** horizontal

3.3 $f(x) = x^2$, $a = 3$, so $f(3) = 9$, $f'(3) = 6$.

(a) Write $L(x)$. [1]

(b) Use it for $x = 3.1$. [1]

(c) State $L(3.1)$. [1]

Solutions

2.1 $L(x) = f(a) + f'(a)(x - a)$.

2.2 $f(4) = 2$; $f'(4) = \frac{1}{4}$.

2.3 the value of $f(x)$ for x near a .

3.1 B.

3.2 B.

3.3 (a) $L(x) = 9 + 6(x - 3)$. (b) $9 + 6(0.1)$. (c) 9.6.