

2.1 Average and Instantaneous Rates of Change

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS instantaneous 瞬时 • difference quotient 差商 • derivative 导数 • slope 斜率

Objective

Build the skills to answer exam questions on **average and instantaneous rates of change at a point**.

You must be able to:

- form the **difference quotient** 差商 for the average rate of change
- describe the instantaneous rate as its limit

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Average and instantaneous rates

Average rate over $[a, a + h]$ is the **secant slope** $\frac{f(a + h) - f(a)}{h}$.

The instantaneous rate at a is the **tangent slope**, obtained as $h \rightarrow 0$.

■ Example

$f(x) = x^2$ at $a = 3$: $\frac{(3 + h)^2 - 9}{h} = 6 + h \rightarrow 6$ as $h \rightarrow 0$.

2 Practice

2.1 State the difference quotient for the average rate over $[a, a + h]$. [1]

2.2 State what happens to it as $h \rightarrow 0$. [1]

2.3 For $f(x) = x^2$, compute $\frac{f(2 + h) - f(2)}{h}$ and simplify. [2]

3 Exam-style questions

3.1 The average rate of change over $[a, a + h]$ is [1]

- A $f(a)h$
- B $\frac{f(a + h) - f(a)}{h}$
- C $f'(a)$
- D h

3.2 The instantaneous rate is the average rate as [1]

- A $h \rightarrow \infty$
- B $h \rightarrow 0$
- C $h = 1$
- D $a \rightarrow 0$

3.3 $f(x) = x^2$ at $a = 3$.

(a) Write $f(3 + h)$. [1]

(b) Form the difference quotient. [1]

(c) State its limit as $h \rightarrow 0$. [1]

Solutions

2.1 $\frac{f(a+h) - f(a)}{h}$.

2.2 it approaches the instantaneous rate of change (the derivative).

2.3 $\frac{(2+h)^2 - 4}{h} = \frac{4h + h^2}{h} = 4 + h$.

3.1 B.

3.2 B.

3.3 (a) $(3+h)^2$. (b) $\frac{(3+h)^2 - 9}{h} = 6 + h$. (c) 6.