

1.6 Determining Limits Using Algebraic Manipulation

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS factor and cancel 因式分解并约分 • indeterminate form 未定式 • limit 极限
• direct substitution 直接代入

Objective

Build the skills to answer exam questions on **determining limits using algebraic manipulation**.

You must be able to:

- recognise an **indeterminate form** 未定式 $\frac{0}{0}$
- factor and cancel to evaluate the limit

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Algebraic manipulation

When direct substitution gives $\frac{0}{0}$, factor and cancel the common factor, then substitute.

■ Example

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{x - 2} = \lim_{x \rightarrow 2} (x + 2) = 4.$$

2 Practice

2.1 State what to do when substitution gives $\frac{0}{0}$. [1]

2.2 Simplify $\frac{x^2 - 9}{x - 3}$. [1]

2.3 Evaluate $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$. [2]

3 Exam-style questions

3.1 If direct substitution gives $\frac{0}{0}$, you should [1]

- **A** conclude the limit is 0
- **B** factor and cancel
- **C** stop — no limit exists
- **D** divide by x

3.2 $\frac{x^2 - 1}{x - 1}$ simplifies to [1]

- **A** $x - 1$
- **B** $x + 1$
- **C** $x^2 + 1$
- **D** 1

3.3 $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$.

(a) Factor the numerator. [1]

(b) Cancel $(x - 1)$. [1]

(c) Substitute $x = 1$. [1]

Solutions

2.1 factor and cancel the common factor, then substitute.

2.2 $x + 3$ (for $x \neq 3$).

2.3 $\lim_{x \rightarrow 3}(x + 3) = 6$.

3.1 B.

3.2 B.

3.3 (a) $(x - 1)(x + 1)$. (b) $x + 1$. (c) 2.