AP CALCULUS AB • EXERCISE SHEET

1.6 Determining Limits Using Algebraic Manipulation

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **determining limits using algebraic manipulation**.

You must be able to:

- recognise an **indeterminate form** 未定式 $\frac{0}{0}$
- factor and cancel to evaluate the limit

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Algebraic manipulation

When direct substitution gives $\frac{0}{0}$, factor and cancel the common factor, then substitute.

Example

$$\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}=\lim_{x\to 2}(x+2)=4.$$

2 Practice

2.1 State what to do when substitution gives $\frac{0}{0}$. [1]

2.2 Simplify $\dfrac{x^2-9}{x-3}$. [1]

2.3 Evaluate $\lim_{x\to 3} \dfrac{x^2-9}{x-3}$. [2]

3 Exam-style questions

3.1 If direct substitution gives $\frac{0}{0}$, you should [1]

- **A** conclude the limit is 0
- **B** factor and cancel
- **C** stop —no limit exists
- **D** divide by x

3.2 $\dfrac{x^2-1}{x-1}$ simplifies to [1]

- **A** $x-1$
- **B** $x+1$
- **C** x^2+1
- **D** 1

3.3 $\lim_{x\to 1} \dfrac{x^2-1}{x-1}$.

(a) Factor the numerator. [1]

(b) Cancel $(x-1)$. [1]

(c) Substitute $x=1$. [1]

4 Go further

- work through the **1.6 Determining Limits Using Algebraic Manipulation** lesson on the **Learn** page;

- read the **Limits and Continuity** section of the AP Calculus AB handout on the **Know** page.

Solutions

2.1 factor and cancel the common factor, then substitute.

2.2 $x + 3$ (for $x \neq 3$).

2.3 $\lim_{x \to 3}(x + 3) = 6$.

3.1 B.

3.2 B.

3.3 (a) $(x - 1)(x + 1)$. (b) $x + 1$. (c) 2.