AP CALCULUS AB • EXERCISE SHEET

1.1 Introducing Calculus: Can Change Occur at an Instant?

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **whether change can occur at an instant**.

You must be able to:

- find an **average rate of change** 平均变化率 over an interval
- describe the **instantaneous rate of change** 瞬时变化率 as a limit of average rates

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Average vs instantaneous rate

The **average rate of change** of f over $[a, b]$ is $\dfrac{f(b) - f(a)}{b - a}$.

The **instantaneous rate** at a point is the **limit** of these average rates as the interval shrinks to zero —this is the idea calculus makes precise.

■ Example

$f(x) = x^2$ on $[1, 3]$: average rate $= \dfrac{9 - 1}{3 - 1} = 4$.

2 Practice

2.1 State the average rate of change of f on $[a, b]$. [1]

2.2 State what the instantaneous rate of change is the limit of. [1]

2.3 For $f(x) = x^2$, find the average rate of change on $[1, 3]$. [2]

3 Exam-style questions

3.1 The instantaneous rate of change is [1]

- **A** the average over $[a, b]$
- **B** the limit of average rates as the interval shrinks
- **C** always zero
- **D** the slope of a single secant line

3.2 The average rate of change of f on $[a, b]$ is [1]

- **A** $f(b) \cdot f(a)$
- **B** $\dfrac{f(b) - f(a)}{b - a}$
- **C** $f(b) + f(a)$
- **D** $b - a$

3.3 $f(x) = x^2$.

(a) Find $f(1)$ and $f(3)$. [1]

(b) Find the average rate of change on $[1, 3]$. [1]

(c) State what a shrinking interval approaches. [1]

4 Go further

- work through the **1.1 Introducing Calculus: Can Change Occur at an Instant?** lesson on the **Learn** page;
- read the **Limits and Continuity** section of the AP Calculus AB handout on the **Know** page.

Solutions

2.1 $\dfrac{f(b)-f(a)}{b-a}$.

2.2 the average rates of change over intervals shrinking to zero.

2.3 $\dfrac{f(3)-f(1)}{3-1}=\dfrac{9-1}{2}=4$.

3.1 **B**.

3.2 **B**.

3.3 (a) $f(1)=1$, $f(3)=9$. (b) $\dfrac{9-1}{2}=4$. (c) the instantaneous rate of change.