Past-paper walk-through

Finding the Average Value of a Function on an Interval ▪ 8.1

eXercise

Questions in this set

- 2025 Q1
- 2024 Q1
- 2023 Q1
- 2022 Q1
- 2021 Q1
- 2019 Q1
- 2018 Q4
- 2016 Q5
- 2015 Q3
- 2014 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(a) Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6 \arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(b) Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \leq t \leq 4$. Show the setup for your calculations.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(c) Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(d) At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.

Solution 1

(a) Mark scheme

- Average value $= \dfrac{1}{4-0}\int_0^4 C(t)\,dt = \dfrac{1}{4}(11.112896) = 2.778224$, so the average number of acres affected from $t = 0$ to $t = 4$ is about 2.778 acres. Point 1: correct integral $\dfrac{1}{4-0}\int_0^4 C(t)\,dt$, with or without dt, with evidence of division by 4. Point 2: correct answer, accurate to 3 decimal places (2.778 or 2.778224), with or without supporting work.

Solution 1

(b) Mark scheme

- Average rate of change of C over $[0,4]$: $\dfrac{C(4)-C(0)}{4-0} = 1.282008$. Set $C'(t) = \dfrac{38}{25+t^2}$ equal to this: $\dfrac{38}{25+t^2} = 1.282008 \Rightarrow t = 2.154298 \approx 2.154$ weeks. Point 3: presents the average rate of change expression or value (e.g. $\dfrac{C(4)-C(0)}{4-0}$, $\dfrac{C(4)}{4}$, $\dfrac{5.128}{4}$, or 1.282). Point 4: correct answer $t \approx 2.154$, supported by the equation $C'(t) =$ average rate of change.

Solution 1

(c)

Mark scheme

- End behavior: $\lim_{t\to\infty} C'(t) = \lim_{t\to\infty} \dfrac{38}{25+t^2} = 0$. Point 5: correct limit expression, either $\lim_{t\to\infty} C'(t)$ or $\lim_{t\to\infty} C(t)$. Point 6: correct value 0 (only earnable if the limit expression used C', not C).

Solution 1

(d)

Mark scheme

- $A'(t) = C'(t) - 0.1 \ln t$. Set $A'(t) = 0$: $C'(t) = 0.1 \ln t \Rightarrow t = 11.441700$. Candidates test: $A(4) = 5.128031$, $A(11.4417) = 7.316978$, $A(36) = 1.743056$. The maximum is at $t = 11.442$ (or 11.441) weeks. Point 7: considers $A'(t) = 0$ (i.e., $C'(t) - 0.1 \ln t = 0$). Point 8: justification via a correct global (candidates-test) argument evaluating A at $t = 4$, $t = 11.4417$, and $t = 36$ (or an equivalent sign-analysis argument that $A'(t) > 0$ then $A'(t) < 0$). Point 9: correct answer $t \approx 11.442$ weeks, supported by work.

Question 1

2024 ▪ Q1

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \leq t \leq 12$, selected values of $C(t)$ are given in the table shown.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

(a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \leq t \leq 7$. Show the work that leads to your answer and include units of measure.

Question 1

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

Question 1

2024 ▪ Q1

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \leq t \leq 12$, selected values of $C(t)$ are given in the table shown.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

(b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t)\,dt$. Interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem.

Question 1

2024 ▪ Q1

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \leq t \leq 12$, selected values of $C(t)$ are given in the table shown.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

(c) For $12 \leq t \leq 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \dfrac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

Question 1

2024 ▪ Q1

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \leq t \leq 12$, selected values of $C(t)$ are given in the table shown.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

(d) For the model defined in part (c), it can be shown that $C''(t) = \dfrac{0.2455e^{0.01t}(100 - t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

Solution 1

(a)

Mark scheme

- $C'(5) \approx \dfrac{C(7) - C(3)}{7 - 3} = \dfrac{69 - 85}{4} = -4$ degrees Celsius per minute. 1 point for the difference-quotient setup with supporting work (a difference over a quotient using table values), 1 point for the correct value with units ('degrees Celsius per minute' attached to the numeric value).

Solution 1

(b) Mark scheme

- Left Riemann sum with the three subintervals from the table: $\int_0^{12} C(t)\,dt \approx (3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7) = 3(100) + 4(85) + 5(69) = 300 + 340 + 345 = 985$. Interpretation: $\frac{1}{12}\int_0^{12} C(t)\,dt$ is the average temperature of the coffee (in degrees Celsius) over the interval $t = 0$ to $t = 12$. 1 point for the correct form of the left Riemann sum (widths times left-endpoint values), 1 point for the numeric estimate 985, 1 point for an interpretation that names both 'average temperature' and the time interval.

Solution 1

(c)

Mark scheme

- $C(20) = C(12) + \int_{12}^{20} C'(t)\,dt = 55 - 14.670812 = 40.329188 \approx 40.329$ degrees Celsius. 1 point for setting up the definite integral of $C'(t)$ with correct limits 12 to 20, 1 point for adding the initial condition $C(12) = 55$ (symbolically or numerically), 1 point for the final numeric answer 40.329 (or 40.330) with supporting work shown.

Solution 1

(d) Mark scheme

- Because $C''(t) > 0$ on $12 < t < 20$ (from $C''(t) = \dfrac{0.2455e^{0.01t}(100-t)}{t^2}$, numerator and denominator both positive on this interval), the rate of change of temperature $C'(t)$ is increasing on this interval — that is, the temperature of the coffee is changing at an increasing rate. The single point requires a correct answer ('increasing rate') together with a reason that references the sign of the second derivative of C (not just evaluation at one point, and not an ambiguous pronoun).

Question 1

2023 ▪ Q1

A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f, where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

t **(seconds)**	**0**	**60**	**90**	**120**	**135**	**150**
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

Question 1

(a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t)\,dt$ in the context of the problem. Use a right Riemann sum with the three subintervals $[60, 90]$, $[90, 120]$, and $[120, 135]$ to approximate the value of $\int_{60}^{135} f(t)\,dt$.

Question 1

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0