

Past-paper walk-through

Volume with Disc Method: Revolving Around Other Axes ■ 8.10

eXercise

Questions in this set

- 2014 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 2

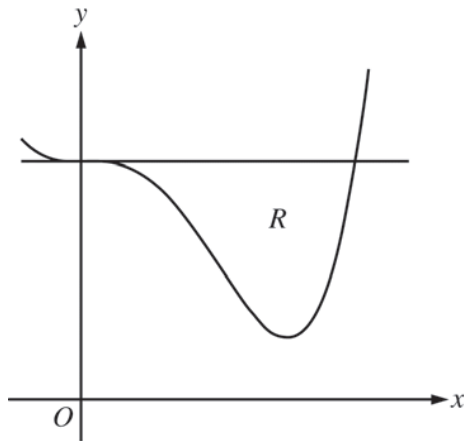
2014 ■ Q2

[Graph of $y = f(x)$ for $x \geq 0$: a curve starting near the origin, dipping down below the x -axis to a minimum, then rising steeply back up and crossing above $y = 0$; the region R is labelled in the dip between the curve and the x -axis/line $y = 4$, roughly between the two crossing points of $f(x) = 4$.]

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

- (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
- (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is an isosceles right triangle with a leg in R . Find the volume of the solid.
- (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k .

Question 2



Solution 2

(a)

Mark scheme

- $f(x) = 4$ at $x = 0$ and $x = 2.3$ (the intersection points bounding R). Volume = $\pi \int_0^{2.3} [(4+2)^2 - (f(x)+2)^2] dx = 98.868$ (or 98.867), using washers with outer radius 6 (from $y = -2$ to $y = 4$) and inner radius $f(x) + 2$ (from $y = -2$ to $y = f(x)$). Points : 2 for the correct integrand, 1 for correct limits of integration (0 to 2.3), 1 for the correct numeric answer

Solution 2

(b)

Mark scheme

- Volume = $\int_0^{2.3} \frac{1}{2}(4-f(x))^2 dx = 3.574$ (or 3.573), using that an isosceles right triangle with legs = 4 — $f(x)$ (the vertical distance in R) has area $(1/2)s^2$. Points :
2 for the correct integrand, 1 for the correct numeric answer.

Solution 2

(c)

Mark scheme

- Set the area of the left piece equal to the area of the right piece:
 $\int_0^k (4 - f(x)) dx = \int_k^{2.3} (4 - f(x)) dx.$ (Equation only – – do not solve for k .) Points :
 1 for a correct expression for the area of one region, 1 for a correct equation setting the two regions' areas