Past-paper walk-through

Reasoning Using Slope Fields ▪ 7.4

eXercise

Questions in this set

- 2026 Q3
- 2023 Q3
- 2022 Q5
- 2015 Q4

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2026 ▪ Q3

A pie is taken from a hot oven and put on a table. The internal temperature of the pie at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

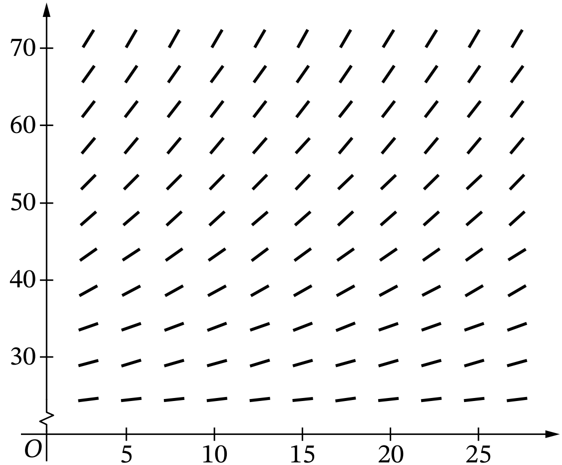
(a) Explain why the following could not be a slope field for the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$.

[Slope field diagram: H-axis (vertical) from 30 to 70+ in increments of 10, t-axis (horizontal) from 0 to 25+ in increments of 5. At every grid point across the full range of t shown, the line segments have the same positive slope for a given horizontal row and the slopes appear to depend only on t (segments in each row are parallel and tilt

Question 3

more steeply positive at greater height H, but they do not appear to flatten toward zero as $H \to 20$ nor do the segments vary with t within a row as required) — segments are drawn as short parallel dashes tilting up-and-to-the-right throughout, uniformly across each horizontal band of H values, without change as t increases.]

Question 3



Question 3

2026 ▪ Q3

A pie is taken from a hot oven and put on a table. The internal temperature of the pie at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

(b) Find the slope of the line tangent to the graph of H at time $t = 0$. Show the work that leads to your answer.

Question 3

2026 ▪ Q3

A pie is taken from a hot oven and put on a table. The internal temperature of the pie at time t minutes can be modeled by the function H that satisfies the differential equation $\dfrac{dH}{dt} = -\dfrac{1}{15}(H - 20)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

(c) It can be shown that $\dfrac{d^2H}{dt^2} = \dfrac{1}{225}(H - 20)$. The line tangent to the graph of H at time $t = 0$ is used to approximate $H(5)$, the internal temperature of the pie at time $t = 5$. Is this approximation an overestimate or an underestimate for the actual value of $H(5)$? Give a reason for your answer.

Question 3

2026 ▪ Q3

A pie is taken from a hot oven and put on a table. The internal temperature of the pie at time t minutes can be modeled by the function H that satisfies the differential equation $\dfrac{dH}{dt} = -\dfrac{1}{15}(H - 20)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

(d) Use separation of variables to find an expression for $H(t)$, the particular solution to the given differential equation with initial condition $H(0) = 75$.

Question 3

2023 ▪ Q3

A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius (°C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\dfrac{dM}{dt} = \dfrac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(a) A slope field for the differential equation $\dfrac{dM}{dt} = \dfrac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.
[Slope field diagram: axes labeled $M(t)$ (vertical, with gridlines at 5 and 70 marked) and t (horizontal, with a gridline at 15 marked), origin O. Short line segments are

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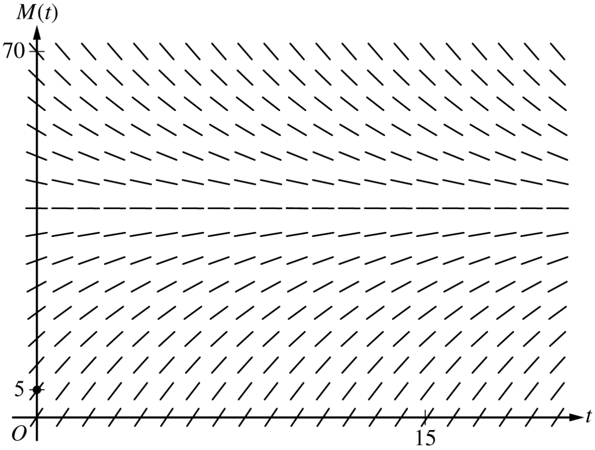
drawn at grid points across the region $0 \leq t \leq 15$-ish, $0 \leq M(t) \leq 70$-ish: segments are nearly horizontal (flat, slightly downward-sloping) near $M(t) = 70$, becoming steeper negative slope moving down toward $M(t) \approx 40$ (segments are horizontal/flat right at $M(t) = 40$), then the slopes become positive and increasingly steep as $M(t)$ decreases from 40 down toward $M(t) = 5$, with the steepest (near-vertical, slope ≈ 1) segments at the bottom of the field near $M(t) = 5$. The point $(0, 5)$ is marked with a star on the vertical axis.]

Question 3



Question 3

2023 ▪ Q3

A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius ($^\circ$C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\dfrac{dM}{dt} = \dfrac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

Question 3

2023 ▪ Q3

A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius ($^\circ$C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\dfrac{dM}{dt} = \dfrac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(c) Write an expression for $\dfrac{d^2M}{dt^2}$ in terms of M. Use $\dfrac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

Question 3

2023 ▪ Q3

A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius ($^\circ$C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

Solution 3

(a)

Mark scheme

- The solution curve through (0,5) starts at (0,5), increases and is concave down, and approaches (curving up toward, but always staying below) the flat slope segments at M=40 as t increases across the shown window; it must not cross any drawn slope-field segments (1 pt: curve passes through (0,5), extends reasonably close to the left and right edges of the given rectangle, has no obvious conflicts with the slope lines, and lies entirely below M=40).

Solution 3

(b) Mark scheme

- $\left.\frac{dM}{dt}\right|_{t=0} = \frac{1}{4}(40-5) = \frac{35}{4}$ (1 pt: value of the derivative at t=0). Tangent line: $y = 5 + \frac{35}{4}(t-0)$, so $M(2) \approx 5 + \frac{35}{4} \cdot 2 = 22.5$; the temperature of the milk at t=2 minutes is approximately 22.5°C (1 pt: approximation using a tangent line through (0,5) with slope 35/4, correctly simplified).

Solution 3

(c) Mark scheme

- $\frac{d^2M}{dt^2} = -\frac{1}{4}\frac{dM}{dt} = -\frac{1}{4}\left(\frac{1}{4}(40 - M)\right) = -\frac{1}{16}(40 - M)$ (1 pt: correct second-derivative expression in terms of M, no subsequent simplification error). Since $M(t) < 40$, $\frac{d^2M}{dt^2} < 0$, so the graph of M is concave down; therefore the tangent-line approximation from part (b) is an overestimate (1 pt: overestimate conclusion with a reason based on concavity/decreasing dM/dt).

Solution 3

(d) Mark scheme

- Separate variables: $\dfrac{dM}{40-M} = \dfrac{1}{4}\,dt$ (1 pt: separates variables). Integrate: $\int \dfrac{dM}{40-M} = \int \dfrac{1}{4}\,dt \Rightarrow -\ln|40-M| = \dfrac{1}{4}t + C$ (1 pt: finds antiderivatives). Apply the initial condition M(0)=5: $-\ln|40-5| = 0 + C \Rightarrow C = -\ln 35$; since M(t)<40, $|40-M| = 40-M$, giving $-\ln(40-M) = \dfrac{1}{4}t - \ln 35$, i.e. $\ln(40-M) = -\dfrac{1}{4}t + \ln 35$ (1 pt: constant of integration correctly found and initial condition used). Solve for M: $40 - M = 35e^{-t/4} \Rightarrow M(t) = 40 - 35e^{-t/4}$ (1 pt: solves for M — the final answer must be exactly this closed form, or equivalent).

Question 5

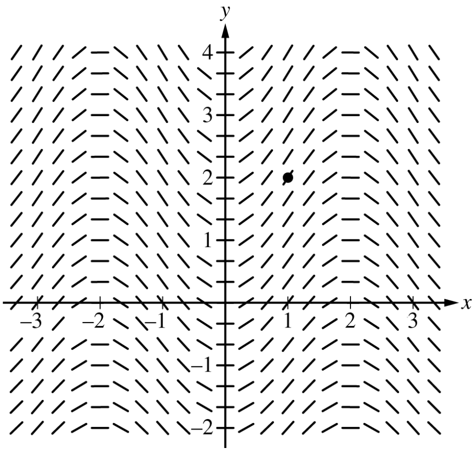
2022 ▪ Q5

Consider the differential equation $\dfrac{dy}{dx} = \dfrac{1}{2}\sin\left(\dfrac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

(a) A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$.

[Graph: a slope field on the region $-3 \leq x \leq 3$, $-2 \leq y \leq 4$ (integer gridlines on both axes). Short line segments at each grid point show the local slope; segments in the left portion (roughly $x < 0$) slant like "/" (positive slope, forward slashes), and segments in the right portion (roughly $x > 0$) slant like "(negative slope, back slashes), with a marked point at $(1, 2)$ where the solution curve should be sketched.]

Question 5



Question 5

2022 ▪ Q5

Consider the differential equation $\dfrac{dy}{dx} = \dfrac{1}{2}\sin\left(\dfrac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

Question 5

2022 ▪ Q5

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

(c) It is known that $f''(x) > 0$ for $-1 \leq x \leq 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

Question 5

2022 ▪ Q5

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

(d) Use separation of variables to find $y = f(x)$, the particular solution to the differential equation

$$\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$$

with the initial condition $f(1) = 2$.

Solution 5

(a)

Mark scheme

- The solution curve must pass through $(1, 2)$, follow the given slope field, satisfy $f(x) > 0$ at every point on the curve, extend reasonably close to the left and right edges of the given square, and have any local maximum/minimum occur only where the slope field shows horizontal segments. This is a graphical sketch, not an algebraic answer; 1 point is earned for a curve satisfying these properties.

Solution 5

(b)

Mark scheme

- $\left.\frac{dy}{dx}\right|_{(x,y)=(1,2)} = \frac{1}{2}\sin\left(\frac{\pi}{2}\right)\sqrt{2+7} = \frac{1}{2}\cdot 1\cdot 3 = \frac{3}{2}$. Tangent line: $y = 2 + \frac{3}{2}(x-1)$. Approximation: $f(0.8) \approx 2 + \frac{3}{2}(0.8-1) = 1.7$. Points: (1) correct tangent line equation (any equivalent form); (2) correct approximation $f(0.8) \approx 1.7$.

Solution 5

(c)

Mark scheme

- Because $f''(x) > 0$ on $-1 \leq x \leq 1$, f is concave up there, so the tangent line lies below the graph of $y = f(x)$ at $x = 0.8$, and the approximation for $f(0.8)$ found in part (b) is an underestimate. 1 point for the correct answer with a reason that references $f''(x) > 0$, f' increasing, or f concave up.

Solution 5

(d) Mark scheme

- $\dfrac{dy}{dx} = \dfrac{1}{2}\sin\left(\dfrac{\pi}{2}x\right)\sqrt{y+7} \Rightarrow \displaystyle\int \dfrac{dy}{\sqrt{y+7}} = \int \dfrac{1}{2}\sin\left(\dfrac{\pi}{2}x\right) dx$. Integrating: $2\sqrt{y+7} = -\dfrac{1}{\pi}\cos\left(\dfrac{\pi}{2}x\right) + C$. Using $f(1) = 2$: $2\sqrt{2+7} = -\dfrac{1}{\pi}\cos\left(\dfrac{\pi}{2}\right) + C \Rightarrow 6 = 0 + C \Rightarrow C = 6$. So $\sqrt{y+7} = 3 - \dfrac{1}{2\pi}\cos\left(\dfrac{\pi}{2}x\right)$, and $y = \left(3 - \dfrac{1}{2\pi}\cos\left(\dfrac{\pi}{2}x\right)\right)^2 - 7$. Points: (1) correct separation of variables; (2) one correct antiderivative; (3) the other correct antiderivative; (4) correct constant of integration found using the initial condition; (5) correctly solves for y.

Question 4

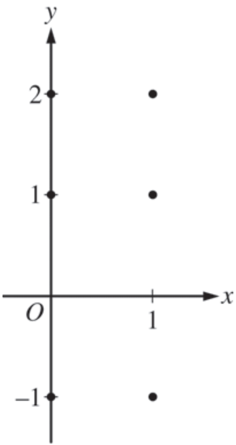
2015 ▪ Q4

Consider the differential equation $\dfrac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

[Slope-field axes labeled y (vertical) and x (horizontal), origin O. Six points are marked with dots, at coordinates $(1, 2)$, $(1, 1)$, $(1, -1)$, and their positions relative to axis tick marks at $y = 2$, $y = 1$, $y = -1$ on the y-axis and $x = 1$ on the x-axis. (The six indicated points are at $x = 1$ paired with $y = 2, 1, -1$, and at $x = 2$ paired with $y = 2, 1, -1$, based on the two columns of dots shown above $y = 2$, $y = 1$, and $y = -1$.)]

Question 4



Question 4

2015 ▪ Q4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

Question 4

2015 ▪ Q4

Consider the differential equation $\dfrac{dy}{dx} = 2x - y$.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

Question 4

2015 ▪ Q4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

Solution 4

(a)

Mark scheme

- Slope field at the six indicated points using $dy/dx = 2x - y$. At $x = 0$: (0,2) slope $= -2$, (0,1) slope $= -1$, (0,−1) slope $= 1$. At $x = 1$: (1,2) slope $= 0$, (1,1) slope $= 1$, (1,−1) slope $= 3$. Points: 1 for correct slope segments drawn at the three points where $x = 0$, 1 for correct slope segments drawn at the three points where $x = 1$. (This is a drawing task — grade by checking the drawn segment directions/steepness at each of the six given points match these computed slopes, not a numeric or algebraic answer.)

Solution 4

(b)

Mark scheme

- $d^2y/dx^2 = 2 - dy/dx = 2 - (2x - y) = 2 - 2x + y$. In Quadrant II, $x < 0$ and $y > 0$, so $2 - 2x + y > 0$ (each term is positive). Therefore all solution curves are concave up in Quadrant II. Points: 1 for the correct expression $d^2y/dx^2 = 2 - 2x + y$, 1 for the correct conclusion (concave up) with a valid reason (sign argument in Quadrant II).

Solution 4

(c)

Mark scheme

- dy/dx at $(x, y) = (2, 3)$ is $2(2) - 3 = 1 \neq 0$. Therefore f has neither a relative minimum nor a relative maximum at $x = 2$, since the derivative is nonzero there. Points: 1 for considering dy/dx at $(2, 3)$, 1 for the correct conclusion (neither min nor max) with justification ($dy/dx \neq 0$ at that point).

Solution 4

(d)

Mark scheme

- If $y = mx + b$, then $dy/dx = d/dx(mx + b) = m$. Substituting into $2x - y = m$ gives $2x - (mx + b) = m$, i.e. $(2 - m)x - (m + b) = 0$ for all x, so $2 - m = 0 \quad m = 2$, and $-(m + b) = 0 \quad b = -m = -2$. Therefore $m = 2$ and $b = -2$. Points: 1 for $d/dx(mx + b) = m$, 1 for the equation $2x - y = m$ (substituting $y = mx + b$ into the differential equation, or equivalent), 1 for the correct answer $m = 2$, $b = -2$.