Exercise walk-through

The Fundamental Theorem of Calculus and Accumulation Functions ▪ 6.4

eXercise

You must be able to

- use $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt = f(x)$ (the **accumulation function 累积函数**)
- apply the **chain rule** when the upper limit is a function of x
- evaluate an accumulation function $g(x) = \displaystyle\int_a^x f(t)\,dt$ at given points

Method — The Fundamental Theorem (derivative form)

If $g(x) = \int_a^x f(t)\,dt$, then $g'(x) = f(x)$. Differentiating "undoes" the integral. For $g(x) = \int_2^x \cos(t^2)\,dt$, we get $g'(x) = \cos(x^2)$ — no integration needed.

Method — Chain rule with a variable upper limit

If the upper limit is $u(x)$, then

$$\frac{d}{dx}\int_a^{u(x)} f(t)\,dt = f\big(u(x)\big)\cdot u'(x).$$

For $\int_0^{x^2} \sin t\,dt$: the answer is $\sin(x^2)\cdot 2x$.

Method — Evaluating an accumulation function

$g(x) = \int_1^x (2t)\,dt$. A value like $g(3) = \int_1^3 2t\,dt = [t^2]_1^3 = 9 - 1 = 8$, and $g(1) = 0$ (zero-width interval).

Method — Sign of the accumulation

g **increases** where $f > 0$ and **decreases** where $f < 0$, because $g' = f$. Reading a graph of f tells you where g rises and falls.

Practice 2.1 ▪ 1 mark

If $g(x) = \int_0^x (t^3 + 1)\,dt$, find $g'(x)$.

Solution 2.1

Worked steps

$g'(x) = x^3 + 1$ **B1**.

Practice 2.2 ▪ 1 mark

Find $\dfrac{d}{dx}\displaystyle\int_1^x \sqrt{t}\,dt$.

Solution 2.2

Worked steps

$\sqrt{x}$ **B1**.

Practice 2.3 ▪ 2 marks

Find $\dfrac{d}{dx}\displaystyle\int_0^{x^3} \cos t \, dt$.

Solution 2.3

Worked steps

$\cos(x^3) \cdot 3x^2$ **M1** **A1** *chain rule.*

Exam-style 3.1 ▪ 1 mark

If $g(x) = \int_a^x f(t)\,dt$, then $g'(x) =$

A $f'(x)$

B $f(x)$

C $f(x) - f(a)$

D $F(x) - F(a)$

Solution 3.1

A $f'(x)$

B $f(x)$ ✓

C $f(x) - f(a)$

D $F(x) - F(a)$

Worked steps

B — by the Fundamental Theorem, $g'(x) = f(x)$.

Exam-style 3.2 ▪ 1 mark

Let $g(x) = \int_2^x f(t)\,dt$, where f is continuous.
(a) Find $g(2)$.
(b) Given $f(5) = 7$, find $g'(5)$.
(c) If $f(t) > 0$ for all $t > 2$, state whether g is increasing or decreasing for $x > 2$, with a reason.

Solution 3.2

Worked steps

(a) $g(2) = 0$ (zero-width interval) **B1**. (b) $g'(5) = f(5) = 7$ **B1**. (c) Increasing — $g'(x) = f(x) > 0$ for $x > 2$ **M1** **A1**.

Exam-style 3.3 ▪ 2 marks

Let $h(x) = \int_0^{2x} e^{-t^2}\,dt$. Find $h'(x)$.

Solution 3.3

Worked steps

$h'(x) = e^{-(2x)^2} \cdot 2 = 2e^{-4x^2}$ **M1** **A1** *chain rule.*