Exercise walk-through

Using the Second Derivative Test to Determine Extrema ▪ 5.7

eXercise

You must be able to

- use f'' at a critical point to classify a **local extremum** 局部极值

Method — Second Derivative Test

At a critical point c where $f'(c) = 0$:

- $f''(c) > 0 \to$ local **minimum** (concave up);
- $f''(c) < 0 \to$ local **maximum** (concave down);
- $f''(c) = 0 \to$ **inconclusive**.

Method — Example

$f(x) = x^2 - 4x$: $f'(x) = 2 > 0$, so the critical point is a local minimum.

Practice 2.1 ▪ 1 mark

State the Second Derivative Test for a local minimum.

Solution 2.1

Method: Second Derivative Test

Worked steps

if $f'(c) = 0$ and $f''(c) > 0$, then c is a local minimum B1.

Practice 2.2 ▪ 2 marks

For $f(x) = x^2 - 4x$ with $f'' = 2$, classify the critical point.

Solution 2.2

Worked steps

$f'' = 2 > 0$, so it is a local minimum M1 A1.

Practice 2.3 ▪ 1 mark

State when the test is inconclusive.

Solution 2.3

Method: Second Derivative Test

Worked steps

when $f''(c) = 0$ B1.

Exam-style 3.1 ▪ 1 mark

If $f'(c) = 0$ and $f''(c) < 0$, then c is a

A local minimum

B local maximum

C inflection point

D nothing

Solution 3.1

A local minimum

B local maximum ✓

C inflection point

D nothing

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

If $f'(c) = 0$ and $f''(c) > 0$, then c is a

A local maximum

B local minimum

C endpoint

D asymptote

Solution 3.2

A local maximum

B local minimum ✓

C endpoint

D asymptote

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$f(x) = x^2 - 6x$, with $f'(3) = 0$.
(a) Find $f''(x)$.
(b) Evaluate $f''(3)$.
(c) Classify $x = 3$.

Solution 3.3

Worked steps

(a) 2 B1. (b) $2 > 0$ B1. (c) local minimum B1.