Past-paper walk-through

Introduction to Related Rates ▪ 4.4

eXercise

Questions in this set

- 2024 Q5
- 2018 Q4

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 5

2024 ▪ Q5

Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\dfrac{dy}{dx} = \dfrac{-2x}{3+4y}$.

(a) There is a point on the curve near $(2, 4)$ with x-coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y-coordinate of this point.

(b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

(c) The curve intersects the positive x-axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

(d) For time $t \geq 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y-coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x-coordinate of the particle's position with respect to time?

Solution 5

(a) Mark scheme

- $\left.\dfrac{dy}{dx}\right|_{(2,4)} = \dfrac{-2(2)}{3+4(4)} = -\dfrac{4}{19}$. Tangent line approximation: $y \approx 4 - \dfrac{4}{19}(3-2) = \dfrac{72}{19}$. 1 point for the correct slope $-4/19$ at $(2,4)$, 1 point for the correct approximation using that slope evaluated at $x = 3$ through the point $(2,4)$.

Solution 5

(b) Mark scheme

- $\dfrac{dy}{dx} = \dfrac{-2x}{3+4y} = 0 \Rightarrow x = 0$, so if $y = 1$ were tangent, the point of tangency would have to be $(0, 1)$. But $(0, 1)$ is not on the curve, because $0^2 + 3(1) + 2(1)^2 = 5 \neq 48$. Therefore the horizontal line $y = 1$ is NOT tangent to the curve. 1 point for considering $dy/dx = 0$ (equivalently $x = 0$, or identifying the point $(0, 1)$), 1 point for the answer 'not tangent' with a valid reason (merely stating '(0,1) is not on the curve' without explanation is insufficient; must show why).

Solution 5

(c) Mark scheme

- At $(\sqrt{48}, 0)$, $\dfrac{dy}{dx} = \dfrac{-2\sqrt{48}}{3+4(0)} = \dfrac{-2\sqrt{48}}{3}$. Since the denominator $3 + 4(0) = 3 \neq 0$, the slope is defined (finite), so the tangent line at this point is NOT vertical. The single point requires clearly showing the slope of the tangent at $(\sqrt{48}, 0)$ is defined and answering 'no' (considering only that the denominator is nonzero is sufficient).

Solution 5

(d) Mark scheme

- Curve: $y^3 + 2xy = 24$. Implicit differentiation with respect to t:
  $3y^2\dfrac{dy}{dt} + 2x\dfrac{dy}{dt} + 2y\dfrac{dx}{dt} = 0$. At $(4, 2)$ with $\dfrac{dy}{dt} = -2$:
  $3(2)^2(-2) + 2(4)(-2) + 2(2)\dfrac{dx}{dt} = 0 \Rightarrow -24 - 16 + 4\dfrac{dx}{dt} = 0 \Rightarrow \dfrac{dx}{dt} = \dfrac{40}{4} = 10.$
  The rate of change of the x-coordinate is 10 units per second. 1 point for attempting implicit differentiation of $y^3 + 2xy = 24$ with respect to t (at most one error), 1 point for the fully correct differentiated equation $3y^2\frac{dy}{dt} + 2x\frac{dy}{dt} + 2y\frac{dx}{dt} = 0$, 1 point for correctly using $dy/dt = -2$ (the y-coordinate is decreasing), 1 point (only earnable with the prior three, no arithmetic mistakes) for the final answer 10.

Question 4

2018 ▪ Q4

t **(years)**	**2**	**3**	**5**	**7**	**10**
$H(t)$ (meters)	1.5	2	6	11	15

The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

Question 4

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

Question 4

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15