Past-paper walk-through

Calculating Higher-Order Derivatives ▪ 3.6

eXercise

Questions in this set

- 2026 Q6
- 2024 Q4
- 2015 Q4
- 2015 Q6

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 6

2026 ▪ Q6

The function f is twice differentiable. The table gives values of f and its derivative f' at selected values of x.

x	**0**	**2**	**3**	**6**
$f(x)$	-1	3	8	5
$f'(x)$	-5	4	9	-2

Question 6

(a) Find $\lim_{x \to 2} \frac{f(x)}{x}$, or state that the limit does not exist.

(b) Let $g(x) = f(f(x))$. Find $g'(2)$. Show the work that leads to your answer.

(c) Let h be a differentiable function such that $h(0) = 10$ and $h'(x) = f'(3x)$. Find $h(2)$. Show the work that leads to your answer.

(d)(i) Let k be the function defined by $k(x) = \int_0^x t^2 f(t)\, dt$. Find $k'(x)$.

(d)(ii) Find $k''(3)$. Show the work that leads to your answer.

Question 6

x	0	2	3	6
$f(x)$	-1	3	8	5
$f'(x)$	-5	4	9	-2

Question 4

2024 ▪ Q4

[Graph of f; x-axis labeled -6 to 7, y-axis labeled -2 to 3; the labeled point $(-6, 0.5)$ is the left endpoint of the curve; the curve rises to a horizontal tangent (peak) at $x = -2$ reaching just above $y = 2$, then descends, crossing the x-axis near $x = 4$, and continues as a straight line down to the labeled point $(6, -1)$ and beyond; the shaded region R lies in the second quadrant between the curve, the vertical line $x = -6$, and the x- and y-axes, spanning roughly $-6 \le x \le 0$.]

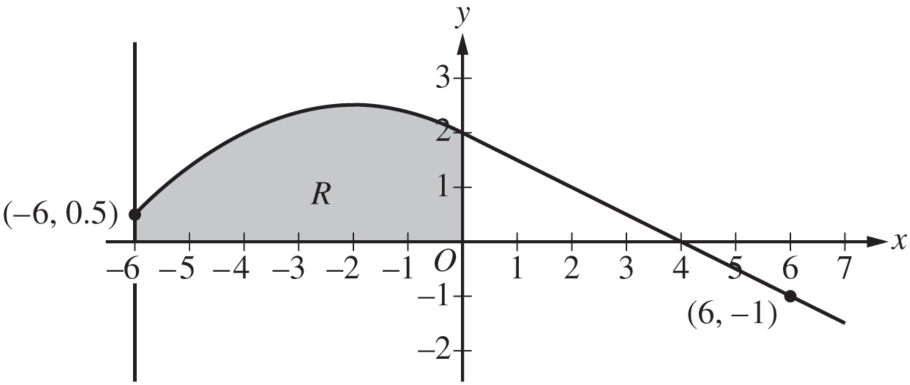
The graph of the differentiable function f, shown for $-6 \le x \le 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \le x \le 7$. Let R be the region in the second quadrant bounded by the graph of f, the vertical line $x = -6$, and the x- and y-axes. Region R has area 12.

(a) The function g is defined by $g(x) = \int_0^x f(t)\,dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.

Question 4

(b) For the function g defined in part (a), find all values of x in the interval $0 \leq x \leq 6$ at which the graph of g has a critical point. Give a reason for your answer.
(c) The function h is defined by $h(x) = \int_{-6}^{x} f'(t)\, dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

Question 4



Graph of f

Solution 4

(a) Mark scheme

- Using the graph of f and $g(x) = \int_0^x f(t)\,dt$:
  $g(-6) = \int_0^{-6} f(t)\,dt = -\int_{-6}^{0} f(t)\,dt = -12$ (since region R, the area under f from -6 to 0, has area 12, given in the problem). $g(4) = \int_0^4 f(t)\,dt = \frac{1}{2}(4)(2) = 4$ (triangle area under the linear piece of f from $x = 0$, where $f(0) = 2$, down to where it crosses the x-axis at $x = 4$).
  $g(6) = \int_0^6 f(t)\,dt = \frac{1}{2}(4)(2) - \frac{1}{2}(2)(1) = 4 - 1 = 3$ (adding the negative triangle from $x = 4$ to $x = 6$, where $f(6) = -1$). 1 point each for $g(-6) = -12$, $g(4) = 4$, $g(6) = 3$ (supporting work not required, but if shown it must be correct to earn the point; unlabeled values are read in the order $g(-6), g(4), g(6)$).

Solution 4

Solution 4

(b)

Mark scheme

- By the Fundamental Theorem of Calculus, $g'(x) = f(x)$. Setting $g'(x) = f(x) = 0$ on $0 \leq x \leq 6$ gives $x = 4$ (where the graph of f crosses the x-axis, matching part (a)). Therefore the graph of g has a critical point at $x = 4$. 1 point for explicitly making the connection $g' = f$, 1 point for the answer $x = 4$ with a valid reason (e.g. $f(4) = 0$ and f changes sign there).

Solution 4

(c) Mark scheme

- $h(x) = \int_{-6}^{x} f'(t)\,dt$, so by the Fundamental Theorem of Calculus $h(6) = \int_{-6}^{6} f'(t)\,dt = f(6) - f(-6) = -1 - 0.5 = -1.5$. $h'(x) = f'(x)$, so $h'(6) = f'(6) = -\dfrac{1}{2}$ (the slope of the linear piece of f on $0 \le x \le 7$, which passes through $(0, 2)$ and $(6, -1)$). $h''(x) = f''(x)$, so $h''(6) = f''(6) = 0$ (since f is linear on $[0, 7]$, its second derivative is 0 there). 1 point for using the Fundamental Theorem of Calculus to evaluate $h(6)$ as $f(6) - f(-6)$, 1 point for the value $h(6) = -1.5$ with supporting work, 1 point for $h'(6) = -1/2$, 1 point for $h''(6) = 0$.

Question 4

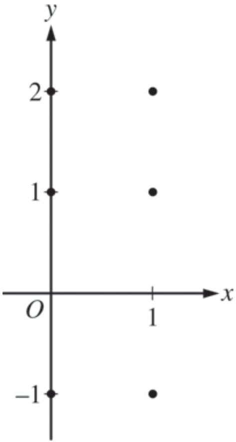
2015 ▪ Q4

Consider the differential equation $\dfrac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

[Slope-field axes labeled y (vertical) and x (horizontal), origin O. Six points are marked with dots, at coordinates $(1, 2)$, $(1, 1)$, $(1, -1)$, and their positions relative to axis tick marks at $y = 2$, $y = 1$, $y = -1$ on the y-axis and $x = 1$ on the x-axis. (The six indicated points are at $x = 1$ paired with $y = 2, 1, -1$, and at $x = 2$ paired with $y = 2, 1, -1$, based on the two columns of dots shown above $y = 2$, $y = 1$, and $y = -1$.)]

Question 4



Question 4

2015 ▪ Q4

Consider the differential equation $\dfrac{dy}{dx} = 2x - y$.

(b) Find $\dfrac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

Question 4

2015 ▪ Q4

Consider the differential equation $\dfrac{dy}{dx} = 2x - y$.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

Question 4

2015 ▪ Q4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

Solution 4

(a)

Mark scheme

- Slope field at the six indicated points using $dy/dx = 2x - y$. At $x = 0$: (0,2) slope $= -2$, (0,1) slope $= -1$, (0,−1) slope $= 1$. At $x = 1$: (1,2) slope $= 0$, (1,1) slope $= 1$, (1,−1) slope $= 3$. Points: 1 for correct slope segments drawn at the three points where $x = 0$, 1 for correct slope segments drawn at the three points where $x = 1$. (This is a drawing task — grade by checking the drawn segment directions/steepness at each of the six given points match these computed slopes, not a numeric or algebraic answer.)

Solution 4

(b)

Mark scheme

- $d^2y/dx^2 = 2 - dy/dx = 2 - (2x - y) = 2 - 2x + y$. In Quadrant II, $x < 0$ and $y > 0$, so $2 - 2x + y > 0$ (each term is positive). Therefore all solution curves are concave up in Quadrant II. Points: 1 for the correct expression $d^2y/dx^2 = 2 - 2x + y$, 1 for the correct conclusion (concave up) with a valid reason (sign argument in Quadrant II).

Solution 4

(c)

Mark scheme

- dy/dx at $(x, y) = (2, 3)$ is $2(2) - 3 = 1 \neq 0$. Therefore f has neither a relative minimum nor a relative maximum at $x = 2$, since the derivative is nonzero there. Points: 1 for considering dy/dx at $(2, 3)$, 1 for the correct conclusion (neither min nor max) with justification ($dy/dx \neq 0$ at that point).

Solution 4

(d)

Mark scheme

- If $y = mx + b$, then $dy/dx = d/dx(mx + b) = m$. Substituting into $2x - y = m$ gives $2x - (mx + b) = m$, i.e. $(2 - m)x - (m + b) = 0$ for all x, so $2 - m = 0 \quad m = 2$, and $-(m + b) = 0 \quad b = -m = -2$. Therefore $m = 2$ and $b = -2$. Points: 1 for $d/dx(mx + b) = m$, 1 for the equation $2x - y = m$ (substituting $y = mx + b$ into the differential equation, or equivalent), 1 for the correct answer $m = 2$, $b = -2$.

Question 6

2015 ▪ Q6

Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\dfrac{dy}{dx} = \dfrac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\dfrac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

Solution 6

(a)

Mark scheme

- dy/dx at (x, y) $= (-1, 1)$ is $1 \,/\, (3(1)^2 - (-1)) = 1/4$. Equation of the tangent line: $y = (1/4)(x + 1) + 1$. Points: 1 for the correct slope 1/4, 1 for the correct tangent line equation.

Solution 6

(b)

Mark scheme

- The tangent line is vertical where the denominator $3y^2 - x = 0$, i.e. $x = 3y^2$. Substituting into $y^3 - xy = 2$: $y^3 - (3y^2)(y) = 2 \quad -2y^3 = 2 \quad y^3 = -1 \quad y = -1$. Then $x = 3y^2 = 3(-1)^2 = 3$ (consistent with the original equation: $(-1)^3 - x(-1) = 2 \quad -1 + x = 2 \quad x = 3$). The tangent line to the curve is vertical at the point $(3, -1)$. Points: 1 for setting $3y^2 - x = 0$, 1 for reducing to an equation in one variable (e.g. $-2y^3 = 2$), 1 for the correct coordinates $(3, -1)$.

Solution 6

(c) Mark scheme

- Differentiating $dy/dx = y/(3y^2 - x)$ implicitly with respect to x gives $d^2y/dx^2 = [(3y^2 - x)(dy/dx) - y(6y(dy/dx) - 1)] / (3y^2 - x)^2$. At $(x, y) = (-1, 1)$ with $dy/dx = 1/4$: $3y^2 - x = 3(1)^2 - (-1) = 4$, so $(3y^2 - x)(dy/dx) = 4 \cdot (1/4) = 1$; $6y(dy/dx) - 1 = 6(1)(1/4) - 1 = 3/2 - 1 = 1/2$, so $y \cdot (6y(dy/dx) - 1) = 1 \cdot (1/2) = 1/2$. Numerator $= 1 - 1/2 = 1/2$. Denominator $= 4^2 = 16$. So $d^2y/dx^2|\_(-1,1) = (1/2)/16 = 1/32$. Points: 2 for correct implicit differentiation to obtain d^2y/dx^2 in terms of x, y, and dy/dx, 1 for substituting $dy/dx = 1/4$ (and $x = -1$, $y = 1$) into that expression, 1 for the correct final numeric answer 1/32.