Past-paper walk-through

Selecting Procedures for Calculating Derivatives ▪ 3.5

eXercise

Questions in this set

- 2015 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2015 ▪ Q2

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.
[Graph showing the curves f and g on axes labeled x and y. Both curves pass through the point $(0, 2)$ on the y-axis and meet again at the point $(2, 4)$. Between these intersection points, one curve rises to a local peak and the other dips to a local trough, crossing each other partway through; the region enclosed on the left (near the peak) is labeled R, and the region enclosed on the right (near the trough) is labeled S. The labeled points are $(0, 2)$ and $(2, 4)$.]

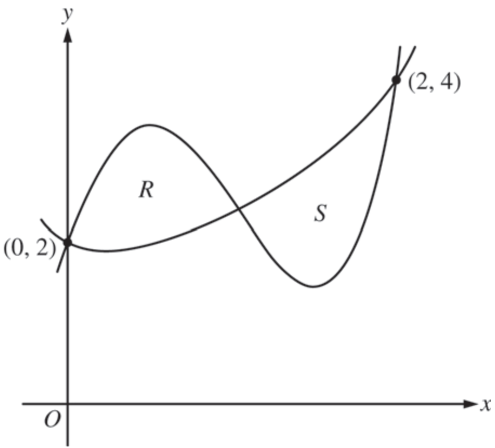
(a) Find the sum of the areas of regions R and S.

(b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

Question 2

(c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

Question 2



Solution 2

(a)

Mark scheme

- The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at $(0, 2)$, $(2, 4)$, and $(A, B) = (1.032832, 2.401108)$. Sum of areas $= \int$ ^A $[g(x) - f(x)]\, dx + \int$ _A^2 $[f(x) - g(x)]\, dx = 0.997427 + 1.006919 = 2.004$. Points: 1 for correct limits (0, A, 2, using the intersection point $A \approx 1.033$), 2 for correct integrands ($[g - f]$ on $[0, A]$ and $[f - g]$ on $[A, 2]$), 1 for the correct final numeric answer 2.004.

Solution 2

(b)

Mark scheme

- Region S is the base of a solid with square cross sections perpendicular to the x-axis, side length $f(x) - g(x)$. Volume $=$ $\int$_A^2 $[f(x) - g(x)]^2\,dx = 1.283$, using $A \approx 1.032832$ as the lower limit (region S spans $x = A$ to $x = 2$). Points: 2 for the correct integrand $[f(x) - g(x)]^2$ with correct limits of integration, 1 for the correct numeric answer 1.283.

Solution 2

(c)

Mark scheme

- $h(x) = f(x) - g(x)$, so $h'(x) = f'(x) - g'(x)$. $h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811). Points: 1 for considering $h'(x) = f'(x) - g'(x)$, 1 for the correct numeric answer ≈ -3.812.