Past-paper walk-through

Connecting Limits at Infinity and Horizontal Asymptotes ▪ 1.15

eXercise

Questions in this set

- 2025 Q1
- 2022 Q6

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(a) Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

Question 1

2025 ▪ Q1

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(b) Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \le t \le 4$. Show the setup for your calculations.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(c) Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

Question 1

2025 ▪ Q1

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(Note: Your calculator should be in radian mode.)

(d) At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.

Solution 1

(a) Mark scheme

- Average value $= \frac{1}{4-0}\int_0^4 C(t)\,dt = \frac{1}{4}(11.112896) = 2.778224$, so the average number of acres affected from $t = 0$ to $t = 4$ is about 2.778 acres. Point 1: correct integral $\frac{1}{4-0}\int_0^4 C(t)\,dt$, with or without dt, with evidence of division by 4. Point 2: correct answer, accurate to 3 decimal places (2.778 or 2.778224), with or without supporting work.

Solution 1

(b) Mark scheme

- Average rate of change of C over $[0, 4]$: $\dfrac{C(4) - C(0)}{4 - 0} = 1.282008$. Set $C'(t) = \dfrac{38}{25 + t^2}$ equal to this: $\dfrac{38}{25 + t^2} = 1.282008 \Rightarrow t = 2.154298 \approx 2.154$ weeks. Point 3: presents the average rate of change expression or value (e.g. $\dfrac{C(4) - C(0)}{4 - 0}$, $\dfrac{C(4)}{4}$, $\dfrac{5.128}{4}$, or 1.282). Point 4: correct answer $t \approx 2.154$, supported by the equation $C'(t) =$ average rate of change.

Solution 1

(c)

Mark scheme

- End behavior: $\lim_{t\to\infty} C'(t) = \lim_{t\to\infty} \frac{38}{25+t^2} = 0$. Point 5: correct limit expression, either $\lim_{t\to\infty} C'(t)$ or $\lim_{t\to\infty} C(t)$. Point 6: correct value 0 (only earnable if the limit expression used C', not C).

Solution 1

(d)

Mark scheme

- $A'(t) = C'(t) - 0.1 \ln t$. Set $A'(t) = 0$: $C'(t) = 0.1 \ln t \Rightarrow t = 11.441700$. Candidates test: $A(4) = 5.128031$, $A(11.4417) = 7.316978$, $A(36) = 1.743056$. The maximum is at $t = 11.442$ (or 11.441) weeks. Point 7: considers $A'(t) = 0$ (i.e., $C'(t) - 0.1 \ln t = 0$). Point 8: justification via a correct global (candidates-test) argument evaluating A at $t = 4$, $t = 11.4417$, and $t = 36$ (or an equivalent sign-analysis argument that $A'(t) > 0$ then $A'(t) < 0$). Point 9: correct answer $t \approx 11.442$ weeks, supported by work.

Question 6

2022 ▪ Q6

Particle P moves along the x-axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.
Particle Q moves along the y-axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \dfrac{1}{t^2}$. At time $t = 1$, the position of particle Q is $y_Q(1) = 2$.

(a) Find $v_P(t)$, the velocity of particle P at time t.

(b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

(c) Find $y_Q(t)$, the position of particle Q at time t.

(d) As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

Solution 6

(a)

Mark scheme

- $v_P(t) = x_P'(t) = 4e^{-t}$. 1 point for the correct answer (an unlabeled response still earns the point; a response that equates $x_P(t)$ with $v_P(t)$ does not).

Solution 6

(b) Mark scheme

- $a_Q(t) = v_Q'(t) = -\dfrac{2}{t^3}$. For $t > 0$, $a_Q(t) < 0$ and $v_Q(t) = \dfrac{1}{t^2} > 0$. Because the velocity and acceleration have opposite signs, the speed of particle Q is decreasing for all $t > 0$. Points: (1) correct expression $a_Q(t) = -2/t^3$; (2) considers/states the signs of $a_Q(t)$ and $v_Q(t)$ (consistent with the presented $a_Q(t)$); (3) correct answer (decreasing for all $t > 0$) with justification that velocity and acceleration have opposite signs.

Solution 6

(c)

Mark scheme

- $y_Q(t) = y_Q(1) + \int_1^t \frac{1}{s^2}\,ds = 2 - \left[\frac{1}{s}\right]_1^t = 2 - \frac{1}{t} + 1 = 3 - \frac{1}{t}$. Points: (1) correct integral expression; (2) uses the initial condition $y_Q(1) = 2$; (3) correct answer, $y_Q(t) = 3 - 1/t$.

Solution 6

(d)

Mark scheme

- For particle P: $\lim_{t\to\infty}(6-4e^{-t})=6$. For particle Q: $\lim_{t\to\infty}\left(3-\frac{1}{t}\right)=3$. Because $6>3$, particle P will eventually be farther from the origin. Points: (1) one correct limit; (2) correct answer with reason based on both limits.