4. Contextual Applications of Differentiation

Starter Quiz ·

AP Calculus AB Name: _____ Score: _____

1. $V(t)$ is water volume in litres at time t minutes. The units of $V'(t)$ are...

   ☐ litres per minute

   ☐ minutes

   ☐ minutes per litre

   ☐ litres

2. Velocity is the derivative of...

   ☐ position

   ☐ speed

   ☐ time

   ☐ acceleration

3. $C(x)$ is cost in dollars for x items. The units of the marginal cost $C'(x)$ are...

   ☐ dollars

   ☐ dollars per item

   ☐ items

   ☐ items per dollar

4. A related-rates problem links the rates of quantities that change with respect to...

   ☐ nothing

   ☐ temperature

   ☐ time

   ☐ each other only

5. With $\dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$, $r = 5$, and $\dfrac{dr}{dt} = 2$, find $\dfrac{dV}{dt}$ as a multiple of π (enter the number before π).

   _____

6. The linearization of f at a is $L(x) =$

   ☐ $f(a) + f'(a)(x - a)$

   ☐ $f'(a) + f(a)(x - a)$

   ☐ $f(a) \cdot f'(a)$

   ☐ $f(x) - f(a)$

7. Assuming the differentiability and derivative-ratio hypotheses hold, L'Hospital's Rule applies to **which** indeterminate forms?

   ☐ $\dfrac{\infty}{\infty}$

   ☐ $\dfrac{3}{0}$

   ☐ $\dfrac{5}{2}$

   ☐ $\dfrac{0}{0}$

   Tick every correct option.

8. If $f'(a) < 0$, the quantity is decreasing at input a.

   ☐ True ☐ False

9. For $v(t) = 3t^2 - 12t + 9$, the object is at rest when $v = 0$. One such time is $t = 1$; find the **other** time.

   _____

10. For $V(t) = 100 - 4t^2$, the rate is $V'(t) = -8t$. Find $V'(3)$ (litres per minute).

    _____

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Answers ·

AP Calculus AB

1. **litres per minute**

   Output per input: litres per minute.

2. **position**

   $v = s'$: velocity is the derivative of position.

3. **dollars per item**

   Output per input: dollars per item.

4. **time**

   The variables are functions of time, linked via the chain rule.

5. **200**

   $4\pi(25)(2) = 200\pi$.

6. $f(a) + f'(a)(x - a)$

   Point-slope of the tangent: $f(a) + f'(a)(x - a)$.

7. $\frac{\infty}{\infty}$; $\frac{0}{0}$

   Only $\frac{0}{0}$ and $\frac{\infty}{\infty}$. $\frac{5}{2}$ is already determinate; $\frac{3}{0}$ is an asymptote, not indeterminate.

8. **True**

   A negative derivative means the quantity is falling.

9. **3**

   $3(t^2 - 4t + 3) = 3(t - 1)(t - 3) = 0 \Rightarrow t = 1$ or $t = 3$.

10. **-24**

    $V'(3) = -8(3) = -24$; draining at 24 L/min.