# 6 The area under a curve – Part 1

Name: _____ Class: _____ Date: _____

## Vocabulary

*Write each word three times.*

| **English** | **Write it 3 times** (English) |
|---|---|
| Riemann sum | |

## Recall

You can find the area of simple shapes:

- The area of a rectangle is width × height.
- But how do you find the area under a **curve**, which has no straight edges?

This sheet introduces the idea behind **integration**. Each idea has a worked example.

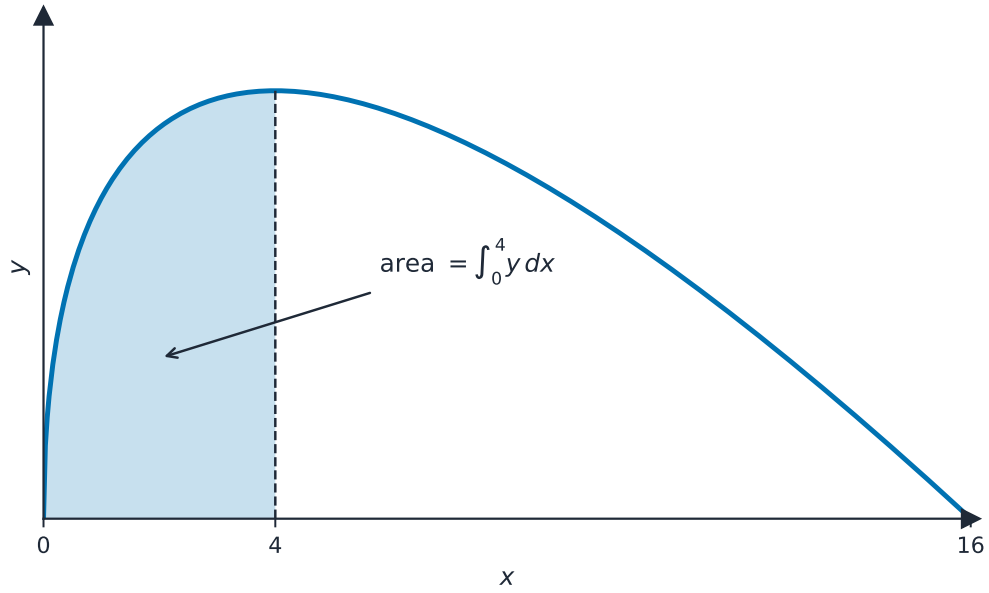
## New ideas

### ■ 1 Area under a curve

The **area under a curve** (between the curve and the $x$-axis) is a useful quantity – for example, the area under a velocity–time graph is the distance travelled.

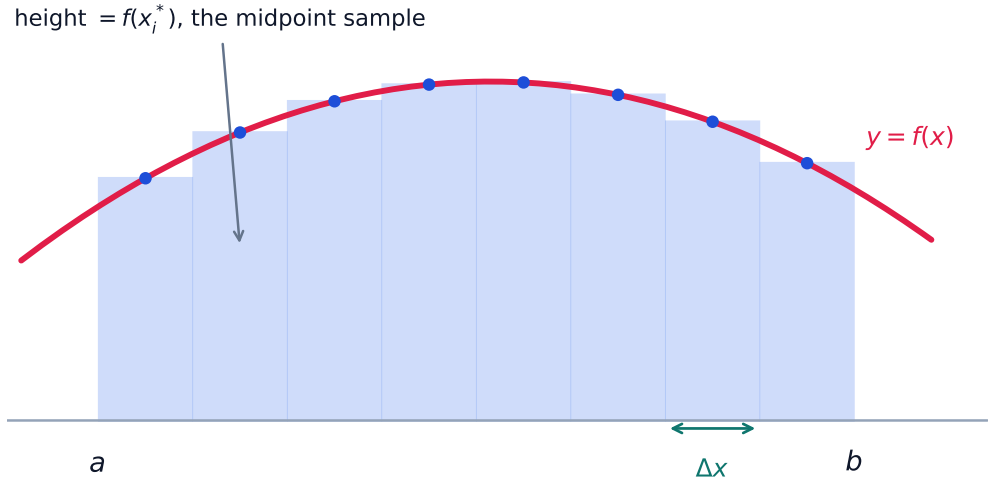


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## 2 Approximating with rectangles

You can estimate that area by covering it with thin **rectangles** and adding their areas – a **Riemann sum** .




$$\sum f(x_i^*)\,\Delta x \to \int_a^b f(x)\,dx \text{ as } n \to \infty$$

## 3 More rectangles, better estimate

The thinner the rectangles (the more you use), the closer the total gets to the true area.

*Worked example.* To estimate the area under $y = x$ from 0 to 4 with rectangles: the exact area is a triangle, $\frac{1}{2} \times 4 \times 4 = 8$, and more rectangles get closer and closer to this.

### ■ 4 The exact area

Making the rectangles infinitely thin gives the **exact** area – this exact value is the **integral**.

**Watch out:** a Riemann sum with a few rectangles is only an **estimate**. The exact area (the integral) is the limit as the rectangles become infinitely thin and infinitely many.

## Practice

Try these in order. The methods are all above.

**2.1** State what "the area under a curve" means. [2]

**2.2** State how a Riemann sum estimates that area. [2]

**2.3** State what happens to the estimate as you use more, thinner rectangles. [1]

**2.4** Find the exact area under $y = x$ from 0 to 6 (it is a triangle). [3]

**2.5** State what the area under a velocity–time graph represents. [1]