

3 The chain rule – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary

Write each word three times.

English

Write it 3 times (English)

chain rule

Recall

In the last topic you used the power rule to differentiate powers of x :

- $\frac{d}{dx} x^3 = 3x^2$.
- But what about something like $(x^2 + 1)^3$ – a function **inside** another function?

This sheet gives the rule for that: the chain rule. Each idea has a worked example.

New ideas

■ 1 Composite functions

A **composite function** is one function tucked **inside** another, like $(x^2 + 1)^3$: the "inside" is $x^2 + 1$ and the "outside" is "cube it".

■ 2 The chain rule

To differentiate a composite function, use the **chain rule** :

differentiate the **outside** (keeping the inside as it is), then **multiply** by the derivative of the inside.

Worked example. For $(x^2 + 1)^3$: the outside gives $3(x^2 + 1)^2$, and the inside $x^2 + 1$ has derivative $2x$, so

$$\frac{d}{dx}(x^2 + 1)^3 = 3(x^2 + 1)^2 \cdot 2x.$$

■ 3 Another example

Worked example. For $(3x + 1)^4$: outside $\rightarrow 4(3x + 1)^3$; inside $3x + 1$ has derivative 3; so the answer is $4(3x + 1)^3 \cdot 3 = 12(3x + 1)^3$.

■ 4 Why “chain”?

The rule links the rates together in a chain: how fast the outside changes with the inside, times how fast the inside changes with x .

Watch out: don't forget to multiply by the derivative of the **inside**. Writing just $3(x^2 + 1)^2$ (and forgetting the $\times 2x$) is the most common chain-rule mistake.

Practice

Try these in order. The methods are all above.

2.1 State, in words, what the chain rule tells you to do. [2]

2.2 For $(x^2 + 1)^3$, state the “inside” function and its derivative. [2]

2.3 Differentiate $(2x + 5)^3$. [3]

2.4 Differentiate $(x^2 + 4)^2$. [3]

2.5 State the most common mistake when using the chain rule.

[1]