

## 6.3 Riemann Sums and Integral Notation

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 11 marks

KEY WORDS definite integral • riemann sum • antiderivative

### Objective

Build the skills to answer exam questions on **Riemann sums and definite-integral notation** – connecting a limit of sums to  $\int_a^b f dx$ .

You must be able to:

- write a Riemann sum with **summation notation**  $\sum$
- recognise the **definite integral** as the limit of Riemann sums as  $n \rightarrow \infty$
- read the meaning of the parts of  $\int_a^b f(x) dx$

### 1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later – follow the steps and you can do the Practice and Exam-style questions yourself.

#### ■ Summation notation for a sum

A right Riemann sum with  $n$  equal pieces on  $[a, b]$  has width  $\Delta x = \frac{b-a}{n}$  and sample points  $x_k = a + k \Delta x$ :

$$\sum_{k=1}^n f(x_k) \Delta x.$$

#### ■ The definite integral as a limit

Letting the rectangles become infinitely thin gives the exact area:

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x.$$

The integral **is** this limit – a sum of infinitely many infinitely thin pieces.

#### ■ Reading the notation

In  $\int_a^b f(x) dx$ :  $a$  and  $b$  are the **limits of integration**,  $f(x)$  is the **integrand**, and  $dx$  marks the variable and the "width" of each piece. The result is a **number** (a signed area), not a function.

#### ■ From a limit back to an integral

A limit like  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \sqrt{1 + \frac{3k}{n}} \cdot \frac{3}{n}$  is a right sum with  $\Delta x = \frac{3}{n}$  on  $[1, 4]$ , so it equals

$$\int_1^4 \sqrt{x} dx.$$

## 2 Practice

Now apply the methods above.

**2.1** For  $\int_2^{10} f dx$  estimated with  $n = 4$  equal subintervals, state  $\Delta x$ . [1]

**2.2** Write, in summation notation, a right Riemann sum for  $\int_0^2 f dx$  with  $n$  equal subintervals. [2]

**2.3** In  $\int_1^5 (3x + 2) dx$ , name the integrand and the limits of integration. [2]

## 3 Exam-style questions

**3.1** The definite integral  $\int_a^b f dx$  is defined as [1]



- A** an antiderivative of  $f$
- B** the limit of Riemann sums as  $n \rightarrow \infty$
- C** the slope of  $f$
- D** the average of  $f(a)$  and  $f(b)$

**3.2** Consider  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{4k}{n}\right)^2 \cdot \frac{4}{n}$ .

(a) State the width  $\Delta x$  and the interval  $[a, b]$ . [2]

(b) Write the limit as a definite integral. [2]

**3.3** A definite integral  $\int_0^6 v(t) dt$  evaluates to a negative number. Explain what a **negative** value tells you about the signed area, in one sentence. [1]

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