AP CALCULUS AB • EXERCISE SHEET

5.6 Concavity and Points of Inflection

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS concave down • concave up • inflection point • concavity

Objective

Build the skills to answer exam questions on **determining concavity**.

You must be able to:

- use the sign of f'' to find **concavity**
- identify an **inflection point**

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

Concavity

- f is **concave up** where $f'' > 0$;
- f is **concave down** where $f'' < 0$.

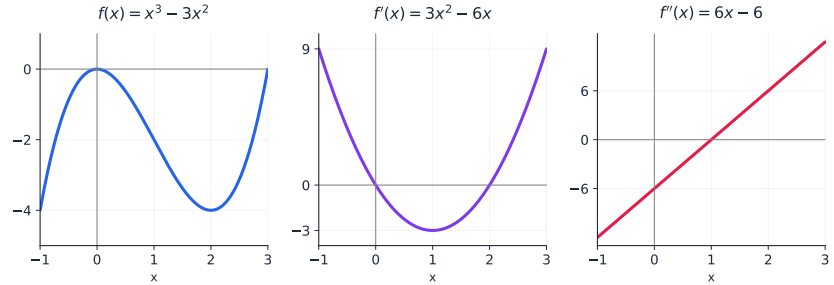
An **inflection point** is a continuous point where concavity changes. Use the signs on neighbouring intervals; the second derivative need not exist at the point itself.

Example

$f(x) = x^3$: $f''(x) = 6x$, which is > 0 for $x > 0$; the inflection point is at $x = 0$.

Worked example: connect the same three graphs

For $f(x) = x^3 - 3x^2$, the derivative is $f'(x) = 3x(x-2)$ and the second derivative is $f''(x) = 6x - 6$.



On $(-1, 0)$, $(0, 2)$ and $(2, 3)$, the signs of f' are positive, negative and positive. Hence f increases, decreases and increases. The sign changes give a local maximum at $(0, 0)$ and minimum at $(2, -4)$. On $(-1, 0)$ the function is increasing but concave down: $f' > 0$ while f' decreases. Concavity changes at 1 because f'' changes from negative to positive; the inflection is $(1, -2)$, where $f'(1) = -3$.

Adding 5 to f leaves both derivatives unchanged. The shape and turning-point x coordinates stay the same; the vertical position needs a given function value.

Worked counterexample: a zero is only a candidate

For $h(x) = x^4$, $h''(x) = 12x^2$ is positive on both sides of 0, so there is no inflection. Yet $h(x) \geq h(0) = 0$, so 0 is a minimum even though $h''(0) = 0$. For $k(x) = x^3$, $k''(x) = 6x$ changes from negative to positive, giving an inflection at 0. Check neighbouring concavity; the second derivative need not exist at the point itself. The function must be continuous at an inflection point.

2 Practice

2.1 State where f is concave up. [1]

2.2 For $f(x) = x^3$, find f'' and where it is concave up. [2]

2.3 State what an inflection point is. [1]

3 Exam-style questions

3.1 f is concave up where [1]

- **A** $f' > 0$
- **B** $f'' > 0$
- **C** $f'' < 0$
- **D** $f = 0$

3.2 An inflection point is where [1]

- **A** $f' = 0$
- **B** concavity changes
- **C** $f = 0$
- **D** f'' is constant

3.3 $f(x) = x^3$.

(a) Find $f''(x)$. [1]

(b) State where $f'' > 0$. [1]

(c) State the inflection point. [1]