

1.15 Connecting Limits at Infinity and Horizontal Asymptotes

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS horizontal asymptote

- ratio of the leading coefficients

- degrees

- limit at infinity

- limit

Objective

Build the skills to answer exam questions on **connecting limits at infinity and horizontal asymptotes**.

You must be able to:

- relate a **limit at infinity** to a **horizontal asymptote**
- find it from the ratio of leading terms

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Limits at infinity

$\lim_{x \rightarrow \infty} f(x) = L$ means $y = L$ is a horizontal asymptote. For a rational function, compare leading terms:

- degree(top) < degree(bottom) $\rightarrow y = 0$;
- degrees equal $\rightarrow y = \frac{\text{leading coeff of top}}{\text{leading coeff of bottom}}$;
- degree(top) > degree(bottom) $\rightarrow$ no horizontal asymptote.

■ Example

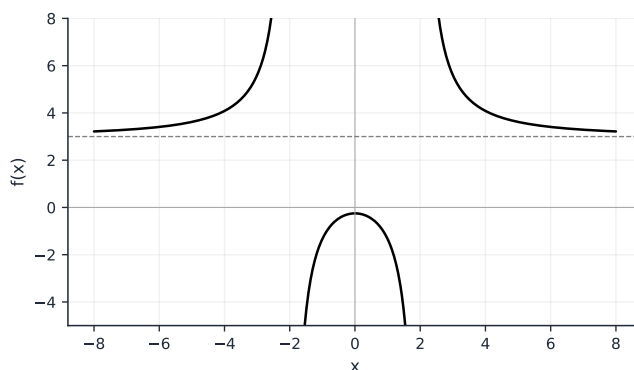
$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$, so $y = 0$ is a horizontal asymptote.

■ **Key knowledge: Read end behaviour from leading powers**

For a rational limit at infinity, divide numerator and denominator by the highest denominator power. Terms containing negative powers then vanish. A horizontal asymptote concerns end behaviour, not every finite input.

■ **Worked example: Read end behaviour from leading powers**

For $f(x) = (3x^2 + 1)/(x^2 - 4)$, division by x^2 gives $(3 + 1/x^2)/(1 - 4/x^2)$. As $x \rightarrow \pm\infty$, its limit is 3. Thus $y = 3$ is a horizontal asymptote at both ends, even though the graph has vertical asymptotes at finite inputs.



2 Practice

2.1 State what $\lim_{x \rightarrow \infty} f(x) = L$ indicates. [1]

2.2 For $f(x) = \frac{1}{x}$, state $\lim_{x \rightarrow \infty} f(x)$. [1]

2.3 For $f(x) = \frac{2x}{x+1}$, state the horizontal asymptote. [2]

3 Exam-style questions

3.1 $\lim_{x \rightarrow \infty} f(x) = L$ means $y = L$ is a [1]

- ☐ **A** vertical asymptote
B horizontal asymptote
C hole
D root

3.2 $\lim_{x \rightarrow \infty} \frac{1}{x}$ equals [1]

- ☐ **A** 1
B 0
C ∞
D -1

3.3 $f(x) = \frac{3x^2}{x^2 + 1}$.

- (a) Compare the degrees. [1]
(b) Take the ratio of the leading coefficients. [1]
(c) State the horizontal asymptote. [1]

3.4 (a) Find $\lim_{x \rightarrow \infty} (2x^2 - x)/(5x^2 + 1)$, showing your algebraic method. [2]

(b) State the horizontal asymptote and explain whether it determines the value at $x = 0$. [2]