# 5.9 Connecting f, f-prime, and f-double-prime

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

**KEY WORDS** local minimum • concave up • decreasing • increasing

## Objective

Build the skills to answer exam questions on **connecting a function, its first derivative, and its second derivative**.

**You must be able to:**

- read $f$, $f'$, and $f''$ together to describe a graph

## 1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

**Connecting $f$, $f'$, $f''$**

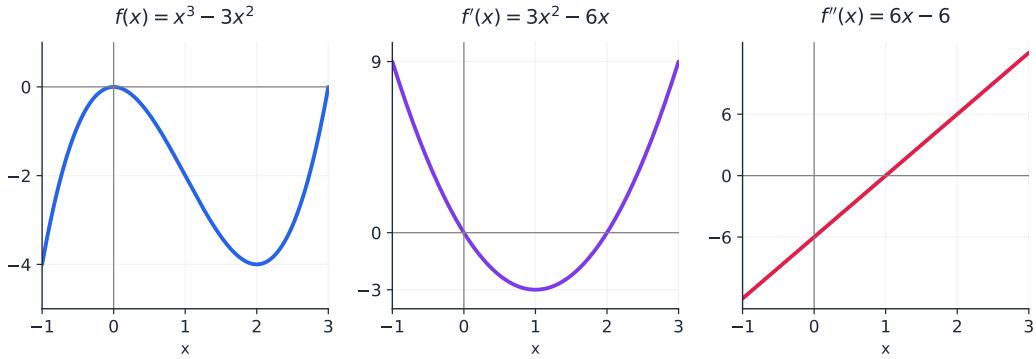
- $f'$ tells increasing/decreasing;
- $f''$ tells concavity;
- $f' = 0$ and $f'' > 0$ are sufficient second-derivative-test conditions for a local **min**, not necessary conditions;
- an **inflection** point is a continuous point where concavity changes; inspect neighbouring intervals.

**Example**

$f(x) = x^3$: $f'(x) = 3x^2$ (zero at 0), $f''(x) = 6x$ (zero at 0).

**Worked example: connect the same three graphs**

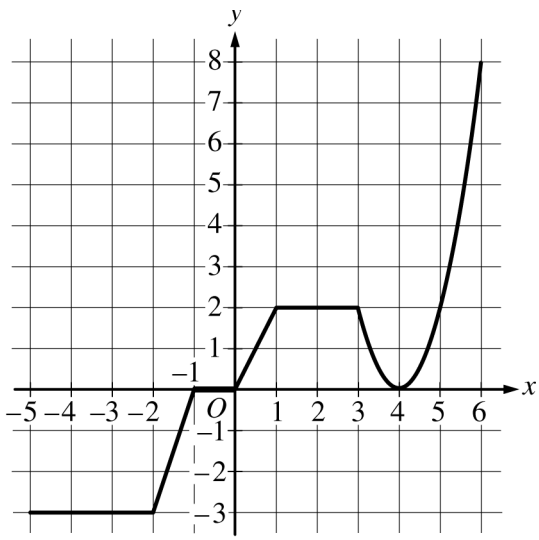
For $f(x) = x^3 - 3x^2$, the derivative is $f'(x) = 3x(x-2)$ and the second derivative is $f''(x) = 6x - 6$.




On $(-1, 0)$, $(0, 2)$ and $(2, 3)$, the signs of $f'$ are positive, negative and positive. Hence $f$ increases, decreases and increases. The sign changes give a local maximum at $(0, 0)$ and minimum at $(2, -4)$. On $(-1, 0)$ the function is increasing but concave down: $f' > 0$ while $f'$ decreases. Concavity changes at 1 because $f''$ changes from negative to positive; the inflection is $(1, -2)$, where $f'(1) = -3$.

Adding 5 to $f$ leaves both derivatives unchanged. The shape and turning-point x coordinates stay the same; the vertical position needs a given function value.

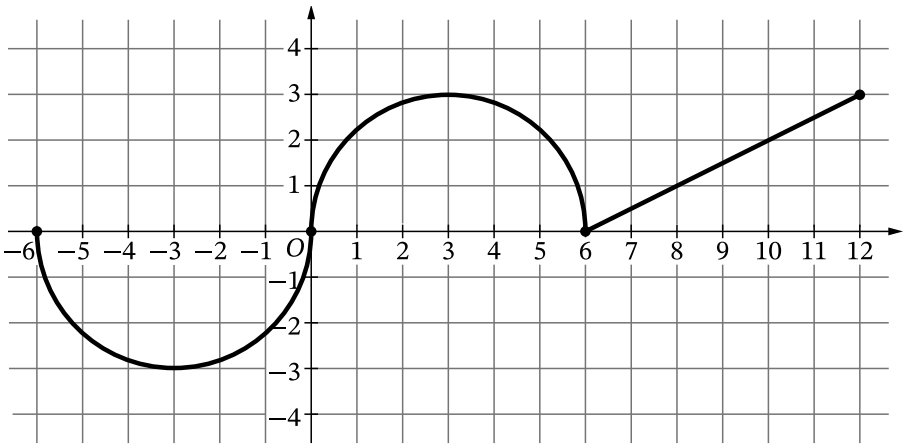
## ■ Original 2018 AB Q 3(c–d): read the displayed derivative




Graph of $g$

The given $g$ is $f'$. For $f$ to be increasing and concave up, $g$ must be positive and increasing: $(0, 1)$ and $(4, 6)$. At 4, $g$ changes from decreasing to increasing, so $f$ has an inflection. Since $g$ is positive on both sides of 4, $f$ has no extremum there. The official scheme awards both the interval answer and its reason.

### ■ Original 2025 AB Q 4B: the notation changes




Graph of $f$

Here $g(x) = \int_6^x f(t)\,dt$, so $g' = f$. The inflections of $g$ occur at $x = -3, 3, 6$: the displayed $f$ changes decreasing/increasing behaviour there. At 0, $f$ increases on both sides, so $g$ remains concave up. At 6 the change of concavity is valid even though $f'$ is undefined at its corner. The official scheme requires evidence tied to the given graph, not simply naming a change in concavity.

### ■ Worked counterexample: a zero is only a candidate

For $h(x) = x^4$, $h''(x) = 12x^2$ is positive on both sides of 0, so there is no inflection. Yet $h(x) \geq h(0) = 0$, so 0 is a minimum even though $h''(0) = 0$. For $k(x) = x^3$, $k''(x) = 6x$ changes from negative to positive, giving an inflection at 0. Check neighbouring concavity; the second derivative need not exist at the point itself. The function must be continuous at an inflection point.

## 2 Practice

**2.1** State what $f''$ tells you about $f$. [1]

**2.2** State sufficient conditions on $f'$ and $f''$ for the second derivative test to prove a local minimum. [1]

**2.3** For $f(x) = x^3$, state where $f' = 0$ and where $f'' = 0$. [2]

## 3 Exam-style questions

**3.1** $f''$ gives information about a graph's [1]

- **A** slope
- **B** concavity
- **C** $x$-intercepts
- **D** domain

**3.2** Which conditions prove a local minimum of $f$ by the second derivative test? [1]

- **A** $f' = 0,\ f'' > 0$
- **B** $f' = 0,\ f'' < 0$
- **C** $f'' = 0$ only
- **D** $f = 0$

**3.3** $f(x) = x^3$.

(a) Find $f'(x)$. [1]

(b) State where $f' = 0$. [1]

(c) State where $f'' = 0$. [1]

## Solutions

Each answer shows the steps, then the **result**.

**2.1**

- its concavity (concave up where $f'' > 0$, down where $f'' < 0$)

**2.2**

- $f' = 0$ and $f'' > 0$

**2.3**

- $f' = 0$ at $x = 0$; $f'' = 0$ at $x = 0$

**3.1 B**

- concavity.

**3.2 A**

- $f' = 0,\ f'' > 0.$

**3.3**

(a)

- $3x^2$

(b)

- $x = 0$

(c)

- $x = 0$