5.8 Sketching a Function and Its Derivatives

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS increasing • concave up • local minimum • decreasing

Objective

Build the skills to answer exam questions on **sketching functions and their derivatives**.

You must be able to:

- relate features of f to the sign and zeros of f'

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

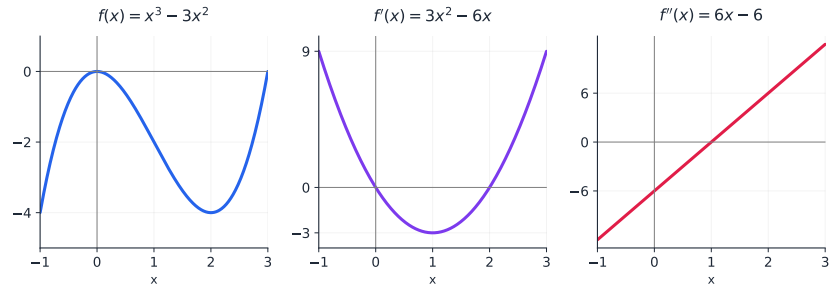
■ Sketching f and f'

- at a differentiable interior local extremum, $f' = 0$; a sign change of f' proves a turn;
- $f' > 0$ on an interval proves that f is increasing there;
- where f is concave up, f' is increasing.

Use these links to sketch one graph from the other. In this sheet's questions, assume differentiability. Sign questions concern open intervals where f' has one constant nonzero sign; each stated extremum has such an interval on either side.

■ Worked example: connect the same three graphs

For $f(x) = x^3 - 3x^2$, the derivative is $f'(x) = 3x(x-2)$ and the second derivative is $f''(x) = 6x - 6$.



On $(-1, 0)$, $(0, 2)$ and $(2, 3)$, the signs of f' are positive, negative and positive. Hence f increases, decreases and increases. The sign changes give a local maximum at $(0, 0)$ and minimum at $(2, -4)$. On $(-1, 0)$ the function is increasing but concave down: $f' > 0$ while f' decreases. Concavity changes at 1 because f'' changes from negative to positive; the inflection is $(1, -2)$, where $f'(1) = -3$.

Adding 5 to f leaves both derivatives unchanged. The shape and turning-point x coordinates stay the same; the vertical position needs a given function value.

2 Practice

2.1 State the value of f' at a local extremum of f. [1]

2.2 State the sign of f' where f is increasing. [1]

2.3 State what f' does where f is concave up. [2]

3 Exam-style questions

3.1 At a local max of f, f' is [1]

☐ **A** positive
B zero
C negative always
D undefined

3.2 Where f is decreasing, its derivative f' is [1]

- **A** positive
- **B** negative
- **C** zero
- **D** increasing

3.3 f has a local minimum at $x = 2$.

(a) State $f'(2)$. [1]

(b) State the sign of f' just left of 2. [1]

(c) State the sign just right of 2. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $f' = 0$

2.2

- positive ($f' > 0$)

2.3

- f' is increasing

3.1 B

- zero.

3.2 B

- negative.

3.3

(a)

- 0

(b)

- negative

(c)

- positive