

5.6 Concavity and Points of Inflection

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS concave down • concave up • inflection point • concavity

Objective

Build the skills to answer exam questions on **determining concavity**.

You must be able to:

- use the sign of f'' to find **concavity**
- identify an **inflection point**

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Concavity

- f is **concave up** where $f'' > 0$;
- f is **concave down** where $f'' < 0$.

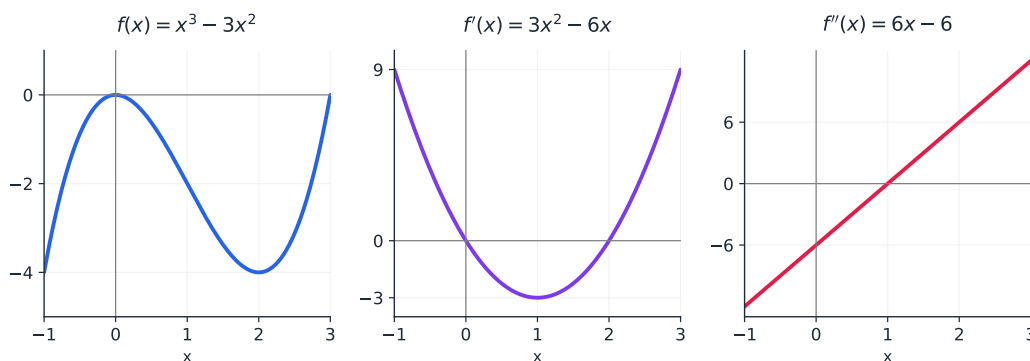
An **inflection point** is a continuous point where concavity changes. Use the signs on neighbouring intervals; the second derivative need not exist at the point itself.

■ Example

$f(x) = x^3$: $f''(x) = 6x$, which is > 0 for $x > 0$; the inflection point is at $x = 0$.

■ Worked example: connect the same three graphs

For $f(x) = x^3 - 3x^2$, the derivative is $f'(x) = 3x(x - 2)$ and the second derivative is $f''(x) = 6x - 6$.



On $(-1, 0)$, $(0, 2)$ and $(2, 3)$, the signs of f' are positive, negative and positive. Hence f increases, decreases and increases. The sign changes give a local maximum at $(0, 0)$ and minimum at $(2, -4)$. On $(-1, 0)$ the function is increasing but concave down: $f' > 0$ while f' decreases. Concavity changes at 1 because f'' changes from negative to positive; the inflection is $(1, -2)$, where $f'(1) = -3$.

Adding 5 to f leaves both derivatives unchanged. The shape and turning-point x coordinates stay the same; the vertical position needs a given function value.

■ **Worked counterexample: a zero is only a candidate**

For $h(x) = x^4$, $h''(x) = 12x^2$ is positive on both sides of 0, so there is no inflection. Yet $h(x) \geq h(0) = 0$, so 0 is a minimum even though $h''(0) = 0$. For $k(x) = x^3$, $k''(x) = 6x$ changes from negative to positive, giving an inflection at 0. Check neighbouring concavity; the second derivative need not exist at the point itself. The function must be continuous at an inflection point.

2 Practice

2.1 State where f is concave up. [1]

2.2 For $f(x) = x^3$, find f'' and where it is concave up. [2]

2.3 State what an inflection point is. [1]

3 Exam-style questions

3.1 f is concave up where

[1]

- ☐ **A** $f' > 0$
B $f'' > 0$
C $f'' < 0$
D $f = 0$

3.2 An inflection point is where

[1]

- ☐ **A** $f' = 0$
B concavity changes
C $f = 0$
D f'' is constant

3.3 $f(x) = x^3$.

(a) Find $f''(x)$.

[1]

(b) State where $f'' > 0$.

[1]

(c) State the inflection point.

[1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- where $f'' > 0$

2.2

- $f''(x) = 6x$; concave up for $x > 0$

2.3

- a point where concavity changes (from up to down or vice versa)

3.1 B

- $f'' > 0$.

3.2 B

- concavity changes.

3.3

(a)

- $6x$

(b)

- $x > 0$

(c)

- $x = 0$