

4.4 Introduction to Related Rates

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS time • related rates

Objective

Build the skills to answer exam questions on **introduction to related rates**.

You must be able to:

- differentiate a relation with respect to **time** to link two rates

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ **Related rates**

When two quantities are related, their rates are related: differentiate the relation with respect to t (using the chain rule).

■ **Example**

$$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}.$$

2 Practice

2.1 State the first step in a related-rates problem. [1]

2.2 Differentiate $A = \pi r^2$ with respect to t . [2]

2.3 State what links the two rates. [1]

3 Exam-style questions

3.1 In related rates, you differentiate the relation with respect to [1]

- ☐ **A** x
B time t
C r
D nothing

3.2 Differentiating $A = \pi r^2$ with respect to t gives [1]

- ☐ **A** $2\pi r$
B $2\pi r \frac{dr}{dt}$
C πr^2
D $2r$

3.3 $V = s^3$ (a cube).

(a) Differentiate with respect to t . [1]

(b) State the chain-rule factor. [1]

(c) Write $\frac{dV}{dt}$. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- write the relation connecting the quantities

2.2

- $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

2.3

- the derivative of the relation with respect to t

3.1 B

- time t .

3.2 B

- $2\pi r \frac{dr}{dt}$.

3.3

(a)

- $\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

(b)

- $\frac{ds}{dt}$

(c)

- $\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$