3.2 Implicit Differentiation

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS implicit differentiation • implicit relation • chain rule

Objective

Build the skills to answer exam questions on **implicit differentiation**.

You must be able to:

- differentiate an **implicit relation** by attaching $\frac{dy}{dx}$ to every y
- solve for $\frac{dy}{dx}$

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

> ■ **Implicit differentiation**
>
> Differentiate both sides with respect to x, writing y' each time you differentiate y (the chain rule), then solve for y'.
>
> ■ **Example**
>
> $x^2 + y^2 = 25 \;\Rightarrow\; 2x + 2y\,y' = 0 \;\Rightarrow\; y' = -\frac{x}{y}$.

2 Practice

2.1 State what to attach when differentiating y. [1]

2.2 Differentiate $x^2 + y^2 = 25$ implicitly. [2]

2.3 State the final step. [1]

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3 Exam-style questions

3.1 In implicit differentiation, differentiating y^2 gives [1]

- ☐ **A** $2y$
- **B** $2y\,y'$
- **C** y'
- **D** $2x$

3.2 After differentiating, you [1]

- ☐ **A** stop
- **B** solve for $\dfrac{dy}{dx}$
- **C** set $y = 0$
- **D** integrate

3.3 $x^2 + y^2 = 25$.

(a) Differentiate both sides. [1]

(b) Isolate the y' term. [1]

(c) Solve for y'. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the factor $\dfrac{dy}{dx}$ (i.e. y')

2.2

- $2x + 2y\,y' = 0$, so $y' = -\dfrac{x}{y}$

2.3

- solve the equation for $\dfrac{dy}{dx}$

3.1 B

- $2y\,y'$.

3.2 B

- solve for $\dfrac{dy}{dx}$.

3.3

(a)

- $2x + 2y\,y' = 0$

(b)

- $2y\,y' = -2x$

(c)

- $y' = -\dfrac{x}{y}$