1.6 Determining Limits Using Algebraic Manipulation

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS factor and cancel • indeterminate form • limit • direct substitution

Objective

Build the skills to answer exam questions on **determining limits using algebraic manipulation**.

You must be able to:

- recognise an **indeterminate form** $\frac{0}{0}$
- factor and cancel to evaluate the limit

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

Algebraic manipulation

When direct substitution gives $\frac{0}{0}$, factor and cancel the common factor, then substitute.

Example

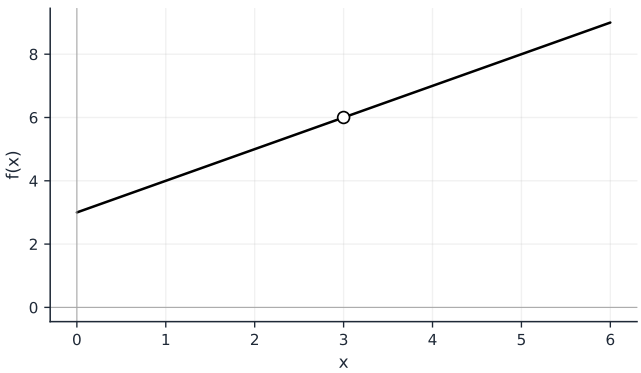
$$\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}=\lim_{x\to 2}(x+2)=4.$$

Key knowledge: Remove an indeterminate form

The form 0/0 identifies a method problem, not a numerical answer. Rewrite the expression for nearby non-target inputs, then evaluate the simplified limit.

Worked example: Remove an indeterminate form

For $f(x)=(x^2-9)/(x-3)$, direct substitution at 3 gives 0/0. Factor and cancel for $x\neq 3$: $f(x)=(x-3)(x+3)/(x-3)=x+3$. Therefore $\lim_{x\to 3} f(x)=6$. Cancellation preserves nearby values; it does not make the original expression defined at 3.



2 Practice

2.1 State what to do when substitution gives $\frac{0}{0}$. [1]

2.2 Simplify $\dfrac{x^2-9}{x-3}$. [1]

2.3 Evaluate $\lim_{x\to 3}\dfrac{x^2-9}{x-3}$. [2]

3 Exam-style questions

3.1 If direct substitution gives $\frac{0}{0}$, you should [1]

- **A** conclude the limit is 0
- **B** factor and cancel
- **C** stop – no limit exists
- **D** divide by x

3.2 $\dfrac{x^2-1}{x-1}$ simplifies to [1]

- **A** $x-1$
- **B** $x+1$
- **C** x^2+1
- **D** 1

3.3 $\lim_{x\to 1} \dfrac{x^2-1}{x-1}$.

(a) Factor the numerator. [1]

(b) Cancel $(x-1)$. [1]

(c) Substitute $x = 1$. [1]

3.4 (a) Rationalise $(\sqrt{x+4}-2)/x$ for $x \neq 0$. [2]

(b) Hence find its limit as $x \to 0$ and explain why direct substitution into the original expression is insufficient. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- factor and cancel the common factor, then substitute

2.2

- $x + 3$ (for $x \neq 3$)

2.3

- $\lim_{x \to 3}(x + 3) = 6$

3.1 B

- factor and cancel.

3.2 B

- $x + 1$.

3.3

(a)

- $(x - 1)(x + 1)$

(b)

- $x + 1$

(c)

- 2

3.4

(a) Multiply by the conjugate: $(\sqrt{x+4} - 2)/x = 1/(\sqrt{x+4} + 2)$ for $x \neq 0$.

(b) The limit is $1/(2+2) = 1/4$; direct substitution into the original expression gives the indeterminate form $0/0$, not its limit.