AP CALCULUS AB • EXERCISE SHEET

1.13 Removing Discontinuities

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS removable discontinuity • hole • limit • continuous • jump

Objective

Build the skills to answer exam questions on **removing discontinuities**.

You must be able to:

- remove a **removable discontinuity** by redefining f at the hole

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

Removing a discontinuity

A removable discontinuity (a hole) can be removed by defining $f(a) = \lim_{x\to a} f(x)$.

Example

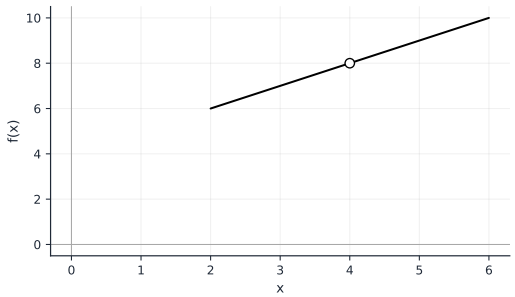
$f(x) = \dfrac{x^2-4}{x-2}$ has a hole at $x = 2$; since the limit there is 4, setting $f(2) = 4$ makes f continuous.

Key knowledge: Repair a removable discontinuity

A removable discontinuity is repaired by assigning the existing finite limit. The simplified nearby formula supplies that limit but need not equal the original value at the missing input.

Worked example: Repair a removable discontinuity

For $f(x) = (x^2-16)/(x-4)$ when $x \neq 4$, simplify to $x+4$. The limit at 4 is 8, so defining $f(4) = 8$ gives a continuous extension. Defining $f(4) = 0$ would leave the same nearby limit but fail continuity.



2 Practice

2.1 State how to remove a removable discontinuity. [1]

2.2 For $f(x) = \dfrac{x^2-4}{x-2}$, find the value that removes the hole at 2. [2]

2.3 State which kind of discontinuity can be removed. [1]

3 Exam-style questions

3.1 A removable discontinuity is removed by setting $f(a)$ equal to [1]

- ☐ **A** 0
- **B** the limit at a
- **C** $f(a+1)$
- **D** ∞

3.2 Which discontinuity can be removed? [1]

- ☐ **A** jump
- **B** infinite
- **C** removable
- **D** none

3.3 $f(x) = \dfrac{x^2 - 9}{x - 3}$, with a hole at 3.

(a) Simplify f. [1]

(b) Find the limit at 3. [1]

(c) State $f(3)$ to remove the hole. [1]

3.4 (a) Find the value needed at 1 to extend $(x^2 - 1)/(x - 1)$ continuously. Show why. [2]

(b) Explain why the same single-point repair cannot remove a jump with left limit 2 and right limit 5. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- define $f(a)$ to equal the limit $\lim_{x \to a} f(x)$

2.2

- the limit is 4, so set $f(2) = 4$

2.3

- a removable discontinuity (a hole)

3.1 B

- the limit at a.

3.2 C

- removable.

3.3

(a)

- $x + 3$ (for $x \neq 3$)

(b)

- 6

(c)

- $f(3) = 6$

3.4

(a) Cancellation gives $x + 1$ near 1, so assign the value 2.

(b) The one-sided limits differ, so no common two-sided limit exists for a point value to match.