# 1.11 Defining Continuity at a Point

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

**KEY WORDS** continuity • continuous • limit • discontinuous

## Objective

Build the skills to answer exam questions on **defining continuity at a point**.

**You must be able to:**

- state the three conditions for **continuity** at a point
- test continuity using $\lim_{x\to a} f(x) = f(a)$

## 1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

### ■ Continuity at a point

$f$ is continuous at $a$ when all three hold:

1. $f(a)$ is defined;
2. $\lim_{x\to a} f(x)$ exists;
3. they are equal: $\lim_{x\to a} f(x) = f(a)$.

### ■ Example

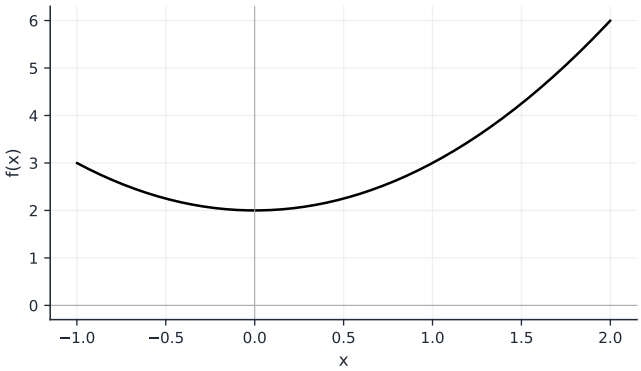
If $\lim_{x\to 2} f(x) = 5$ and $f(2) = 5$, then $f$ is continuous at 2.

### ■ Key knowledge: Test all continuity conditions

Continuity at a requires a defined point value, an existing two-sided limit and equality between them. Finding only one of these is incomplete.

### ■ Worked example: Test all continuity conditions

Let $f(x) = x^2 + 2$ for $x \neq 1$ and $f(1) = k$. The limit at 1 is $1^2 + 2 = 3$. Continuity therefore requires $k = 3$. With that assignment the value exists, the two-sided limit exists, and the two are equal.




## 2 Practice

**2.1** State the three conditions for continuity at $a$. [1]

**2.2** State the single equation summarizing continuity at $a$. [1]

**2.3** If $\lim_{x\to 2} f(x) = 5$ and $f(2) = 5$, is $f$ continuous at 2? [2]

## 3 Exam-style questions

**3.1** $f$ is continuous at $a$ if and only if [1]

- ☐ **A** $f(a)$ is defined
- **B** $\lim_{x\to a} f(x) = f(a)$
- **C** the limit exists only
- **D** $f$ is linear

**3.2** If $\lim_{x\to a} f(x)$ exists but $\neq f(a)$, then $f$ is [1]

- ☐ **A** continuous at $a$
- **B** discontinuous at $a$
- **C** linear
- **D** undefined everywhere

**3.3** $f(2) = 5$ and $\lim_{x \to 2} f(x) = 5$.

(a) Is $f(2)$ defined? [1]

(b) Does the limit exist? [1]

(c) Is $f$ continuous at 2? [1]

**3.4** Let $g(x) = x^2 - x + 1$ for $x \neq 2$ and $g(2) = k$. (a) Find the limit at 2 and the value of $k$ required for continuity. [2]

(b) State the three continuity conditions and explain why your assignment meets them. [2]

## Solutions

Each answer shows the steps, then the **result**.

**2.1**

- $f(a)$ is defined; the limit exists; the limit equals $f(a)$

**2.2**

- $\lim_{x \to a} f(x) = f(a)$

**2.3**

- yes – the limit exists, $f(2)$ is defined, and they are equal

**3.1 B**

- $\lim_{x \to a} f(x) = f(a)$.

**3.2 B**

- discontinuous at $a$.

**3.3**

(a)

- yes

(b)

- yes

(c)

- yes

**3.4**

(a) Limit $= 4 - 2 + 1 = 3$; $k = 3$.

(b) The point value and limit both exist and are equal at 3.