AP CALCULUS AB • EXERCISE SHEET

1.10 Exploring Types of Discontinuities

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS discontinuity • infinite discontinuity • jump discontinuity • removable discontinuity • jump

Objective

Build the skills to answer exam questions on **types of discontinuities**.

You must be able to:

- classify a **discontinuity** as removable, jump, or infinite

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

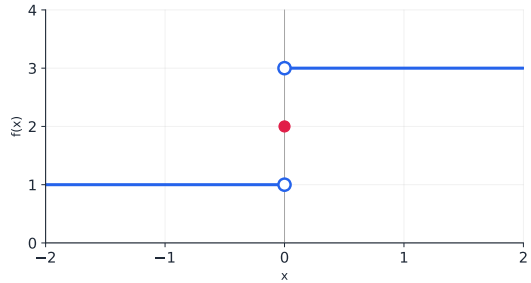
Three types of discontinuity

- **removable** (a **hole**): the limit exists but $\neq f(a)$, or $f(a)$ is undefined;
- **jump**: the left and right limits differ;
- **infinite**: a vertical asymptote (the function blows up).

Worked comparison: classify and justify a repair

At 2, $A(x) = (x^2 - 4)/(x - 2)$ simplifies to $x + 2$ for $x \neq 2$. Its limit is 4; define $A(2) = 4$ to remove the hole. In contrast, $B(x) = 1$ for $x < 2$ and 3 for $x \geq 2$ has unequal finite one-sided limits 1 and 3. Changing $B(2)$ cannot remove its jump. For $C(x) = 1/(x - 2)$, the left side tends to $-\infty$ and the right to $+\infty$: an infinite discontinuity, with no finite limit to use as a repair.

The separate graph below has its jump at $x = 0$: the left-hand limit is 1, the right-hand limit is 3, and $f(0) = 2$. Changing that filled point cannot make the nearby heights agree.



2 Practice

2.1 Name the three types of discontinuity. [1]

2.2 State which type is a "hole". [1]

2.3 State which type occurs at a vertical asymptote. [2]

3 Exam-style questions

3.1 A discontinuity where the left and right limits differ is a [1]

☐ **A** removable discontinuity
B jump discontinuity
C infinite discontinuity
D point of continuity

3.2 A hole in a graph is a [1]

☐ **A** jump discontinuity
B removable discontinuity
C infinite discontinuity
D point of continuity

3.3 Classify each.

(a) A vertical asymptote. [1]

(b) A single missing point. [1]

(c) A step where the graph jumps. [1]

3.4 (a) For $f(x) = (x^2 - 4)/(x - 2)$, identify the discontinuity at 2 and give a point value that removes it. [2]

(b) A function approaches 1 from the left of 3 and 4 from the right. Classify the discontinuity and explain whether changing $f(3)$ can remove it. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- removable, jump, and infinite

2.2

- removable

2.3

- infinite

3.1 B

- jump discontinuity.

3.2 B

- removable discontinuity.

3.3

(a)

- infinite

(b)

- removable

(c)

- jump

3.4

(a) Removable; define $f(2) = 4$.

(b) Jump discontinuity; no single point value can make the unequal one-sided limits agree.