

Exercise walk-through

8.9

Volume with Disc Method: Revolving Around the x- or y-Axis

AP Calculus AB

Volumes with known cross sections

If a solid has a known **cross section** perpendicular to an axis, the volume is the integral of its **area**:

$$V = \int_a^b A(x) dx.$$

The exam phrase “the base of a solid... cross sections are squares” signals exactly this.

square of side s : $A = s^2$

rectangle, height ks : $A = ks^2$

equilateral **triangle**: $A = \frac{\sqrt{3}}{4}s^2$

semicircle, diameter s : $A = \frac{\pi}{8}s^2$

Set the shape's key length equal to the slice length $s(x) = f(x) - g(x)$, then integrate.

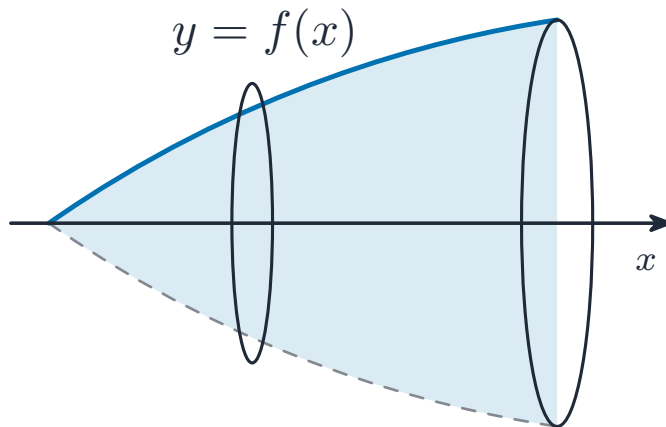
Volumes of revolution – the disc method

Revolving a region about an axis makes a **solid of revolution**. If the region **touches** the axis, each slice is a **disc** :

$$V = \pi \int_a^b [r(x)]^2 dx.$$

About the y-axis: express x as a function of y and integrate in y .

Volume with the Disc Method (About the x - or y -Axis)



rotate the region around the x -axis to sweep a solid

Rotating a region about an axis sweeps out a solid of revolution

Exercises 2.1·2.3

2.1 Write the disc-method integral for revolving $y = x$ on $[0, 2]$ about the x -axis. [1]

2.2 Evaluate $\pi \int_0^2 x^2 dx$. [2]

2.3 Find the volume when $y = \sqrt{x}$ on $[0, 9]$ is revolved about the x -axis. [3]

Exercises 2.1·2.3

2.1 Write the disc-method integral for revolving $y = x$ on $[0, 2]$ about the x -axis. [1]

2.1

- $\pi \int_0^2 x^2 dx$

2.2 Evaluate $\pi \int_0^2 x^2 dx$. [2]

2.2

- $\pi \left[\frac{x^3}{3} \right]_0^2 = \frac{8\pi}{3}$

2.3 Find the volume when $y = \sqrt{x}$ on $[0, 9]$ is revolved about the x -axis. [3]

2.3

- $\pi \int_0^9 x dx = \pi \left[\frac{x^2}{2} \right]_0^9 = \frac{81\pi}{2}$

Exercise 3.1

3.1 Revolving $y = f(x)$ about the x -axis gives the volume

[1]



A $\int_a^b f(x) \, dx$

B $\pi \int_a^b f(x) \, dx$

C $\pi \int_a^b (f(x))^2 \, dx$

D $2\pi \int_a^b f(x) \, dx$

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D $2\pi \int_a^b f(x) \, dx$

3.1 C

- the disc method integrates $\pi(\text{radius})^2$

Exercise 3.2 (a)

3.2 The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the x -axis.

(a) Write the volume integral.

[2]

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(a) Write the volume integral.

[2]

(a)

- $$\pi \int_0^2 (x^2)^2 dx = \pi \int_0^2 x^4 dx$$

Exercise 3.2 (b)

3.2 The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the x -axis.

(b) Evaluate it.

[3]

Exercise 3.2 (b)

3.2 The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the x -axis.

(b) Evaluate it.

[3]

(b)

- $\pi \left[\frac{x^5}{5} \right]_0^2 = \frac{32\pi}{5}$

Exercise 3.3

3.3 The region bounded by $x = y^2$, $y = 0$ to $y = 2$, and the y -axis is revolved about the y -axis. Find the volume. [4]

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3.3

- radius = $x = y^2$; $V = \pi \int_0^2 (y^2)^2 dy = \pi \int_0^2 y^4 dy = \pi \left[\frac{y^5}{5} \right]_0^2 = \frac{32\pi}{5}$