

Exercise walk-through

8.8

# Volumes with Cross Sections: Triangles and Semicircles

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AP Calculus AB

## You must be able to

- use  $A = \frac{\sqrt{3}}{4}s^2$  for an **equilateral triangle** slice
- use  $A = \frac{\pi}{8}s^2$  for a **semicircular** slice (diameter =  $s$ )
- substitute the area formula and integrate

1

The method

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## Method — Equilateral-triangle cross sections

A slice that is an equilateral triangle with side  $s$  has area  $A = \frac{\sqrt{3}}{4}s^2$ . If  $s = f(x) - g(x)$ , then

$$V = \int_a^b \frac{\sqrt{3}}{4} (f(x) - g(x))^2 dx.$$

## Exercises 2.1-2.3

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**2.1** State the area of an equilateral triangle with side  $s$ .

[1]

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**2.2** State the area of a semicircle with diameter  $s$ .

[1]

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**2.3** A solid has equilateral-triangle cross sections with side  $s = x$  on  $[0, 2]$ . Find its volume.

[3]

## Exercises 2.1·2.3

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**2.1** State the area of an equilateral triangle with side  $s$ .

[1]

**2.1**

- $A = \frac{\sqrt{3}}{4}s^2$

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**2.2** State the area of a semicircle with diameter  $s$ .

[1]

**2.2**

- $A = \frac{\pi}{8}s^2$

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**2.3** A solid has equilateral-triangle cross sections with side  $s = x$  on  $[0, 2]$ . Find its volume.

[3]

**2.3**

- $V = \int_0^2 \frac{\sqrt{3}}{4}x^2 dx = \frac{\sqrt{3}}{4} \left[ \frac{x^3}{3} \right]_0^2 = \frac{\sqrt{3}}{4} \cdot \frac{8}{3} = \frac{2\sqrt{3}}{3}$

## Exercise 3.1

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**3.1** A semicircular cross section with diameter  $s$  has area

[1]



- A**  $\frac{\pi}{2}s^2$   
**B**  $\frac{\pi}{4}s^2$   
**C**  $\frac{\pi}{8}s^2$   
**D**  $\pi s^2$

## Exercise 3.1

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**3.1** A semicircular cross section with diameter  $s$  has area

[1]



**A**  $\frac{\pi}{2}s^2$

**B**  $\frac{\pi}{4}s^2$

**C**  $\frac{\pi}{8}s^2$

**D**  $\pi s^2$

**3.1 C**

- radius  $s/2$  gives  $\frac{1}{2}\pi(s/2)^2 = \frac{\pi}{8}s^2$

## Exercise 3.2 (a)

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**3.2** The base of a solid is bounded by  $y = 2 - x$  and the axes in the first quadrant. Cross sections perpendicular to the  $x$ -axis are equilateral triangles.

(a) Write the side length  $s$  as a function of  $x$ .

[1]

## Exercise 3.2 (a)

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**3.2** The base of a solid is bounded by  $y = 2 - x$  and the axes in the first quadrant. Cross sections perpendicular to the  $x$ -axis are equilateral triangles.

(a) Write the side length  $s$  as a function of  $x$ . [1]

(a)

- $s = 2 - x$

## Exercise 3.2 (b)

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**3.2** The base of a solid is bounded by  $y = 2 - x$  and the axes in the first quadrant. Cross sections perpendicular to the  $x$ -axis are equilateral triangles.

(b) Set up the volume integral.

[2]

## Exercise 3.2 (b)

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**3.2** The base of a solid is bounded by  $y = 2 - x$  and the axes in the first quadrant. Cross sections perpendicular to the  $x$ -axis are equilateral triangles.

(b) Set up the volume integral.

[2]

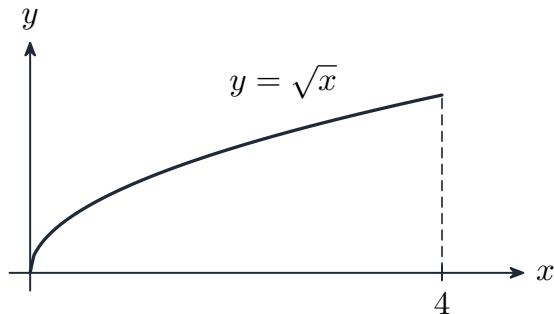
(b)

- $\int_0^2 \frac{\sqrt{3}}{4} (2 - x)^2 dx$

## Exercise 3.3

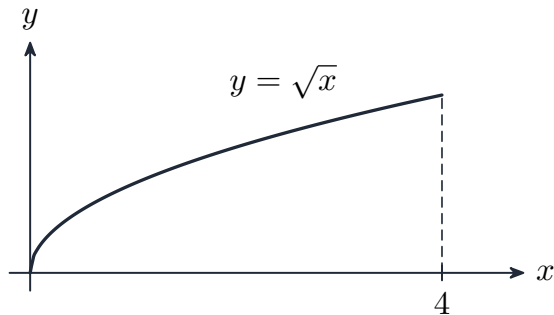
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**3.3** The base is the region between  $y = \sqrt{x}$  and the  $x$ -axis on  $[0, 4]$ . Cross sections perpendicular to the  $x$ -axis are semicircles with diameter across the region. Set up and evaluate the volume. [4]



## Exercise 3.3

**3.3** The base is the region between  $y = \sqrt{x}$  and the  $x$ -axis on  $[0, 4]$ . Cross sections perpendicular to the  $x$ -axis are semicircles with diameter across the region. Set up and evaluate the volume. [4]



**3.3**

- $s = \sqrt{x}; V = \int_0^4 \frac{\pi}{8} (\sqrt{x})^2 dx = \frac{\pi}{8} \int_0^4 x dx = \frac{\pi}{8} \left[ \frac{x^2}{2} \right]_0^4 = \frac{\pi}{8} \cdot 8 = \pi$