

Exercise walk-through

8.10

Volume with Disc Method: Revolving Around Other Axes

AP Calculus AB

Volumes of revolution – the disc method

Revolving a region about an axis makes a **solid of revolution**. If the region **touches** the axis, each slice is a **disc** :

$$V = \pi \int_a^b [r(x)]^2 dx.$$

About the y -axis: express x as a function of y and integrate in y .

Exam tips

- Area between curves is $\int (\text{top} - \text{bottom}) dx$ – find the intersection points for the limits and keep top minus bottom.
- For a **volume of revolution**, add up disc/washer cross-sections of area πr^2 (or $\pi(R^2 - r^2)$).
- The **average value** of f on $[a, b]$ is $\frac{1}{b-a} \int_a^b f dx$.
- Accumulated change is $\int$ of a rate: total = initial value + $\int_a^b (\text{rate}) dt$.
- Integration means “adding up infinitely many tiny pieces” – set up the integrand as one thin slice.

Unit 8 – key takeaways

1

Average value

$\frac{1}{b-a} \int_a^b f$ – not the average rate of change

2

Motion

displacement integrates v ;
distance integrates $|v|$

3

Net change

final = initial + $\int$ rate – watch signs and units

4

Area & volume

top minus bottom; $V = \int A(x) dx$;
discs use πr^2

Exercises 2.1-2.3

2.1 A curve $y = f(x)$ is revolved about $y = 3$. Write the radius. [1]

2.2 Revolving $y = 2$ (constant) region about $y = -1$: state the radius. [1]

2.3 Expand $(\sqrt{x} + 1)^2$. [1]

Exercises 2.1-2.3

2.1 A curve $y = f(x)$ is revolved about $y = 3$. Write the radius. [1]

2.1

- $R = |f(x) - 3|$

2.2 Revolving $y = 2$ (constant) region about $y = -1$: state the radius. [1]

2.2

- $R = 2 - (-1) = 3$

2.3 Expand $(\sqrt{x} + 1)^2$. [1]

2.3

- $x + 2\sqrt{x} + 1$

Exercise 3.1

3.1 Revolving $y = f(x)$ about the line $y = k$ uses radius

[1]

☐

A $f(x)$

B $f(x) - k$

C k

D $f(x) + k$ always

Exercise 3.1

3.1 Revolving $y = f(x)$ about the line $y = k$ uses radius

[1]

☐

A $f(x)$

B $f(x) - k$

C k

D $f(x) + k$ always

3.1 B

- the radius is the distance $f(x) - k$ to the line

Exercise 3.2 (a)

3.2 The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the line $y = -1$.

(a) Write the radius $R(x)$.

[1]

Exercise 3.2 (a)

3.2 The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the line $y = -1$.

(a) Write the radius $R(x)$.

[1]

(a)

- $R = x^2 - (-1) = x^2 + 1$

Exercise 3.2 (b)

3.2 The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the line $y = -1$.

(b) Set up the volume integral.

[2]

Exercise 3.2 (b)

3.2 The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the line $y = -1$.

(b) Set up the volume integral.

[2]

(b)

- $\pi \int_0^2 (x^2 + 1)^2 dx$

Exercise 3.3

3.3 The region under $y = \sqrt{x}$ on $[0, 4]$ is revolved about the line $y = 2$. Set up the volume integral for the solid (radius $= 2 - \sqrt{x}$). [3]

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3.3

- $R = 2 - \sqrt{x}; V = \pi \int_0^4 (2 - \sqrt{x})^2 dx$