

Exercise walk-through

6.8

Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation

AP Calculus AB

Antiderivatives and indefinite integrals

An **indefinite integral** is the **family** of all antiderivatives:

$$\int f(x) dx = F(x) + C.$$

Never drop the **constant of integration**.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{1}{x} dx = \ln |x| + C$$

$$\int e^x dx = e^x + C$$

$$\int \cos x dx = \sin x + C$$

u -substitution – reversing the chain rule

Choose an inside function $u = g(x)$, so $du = g'(x) dx$:

$$\int f(g(x)) g'(x) dx = \int f(u) du.$$

Look for a function **and its derivative** both present.

Evaluate $\int 2x \cos(x^2) dx$

Inside is $u = x^2$, and $du = 2x dx$ is **already there**:

$$\int \cos u du = \sin u + C = \sin(x^2) + C.$$

Spotting that $2x$ is exactly $\frac{du}{dx}$ is the whole trick.

For a **definite** integral: change the limits to u -values, or convert back to x first.

Selecting Techniques for Antidifferentiation

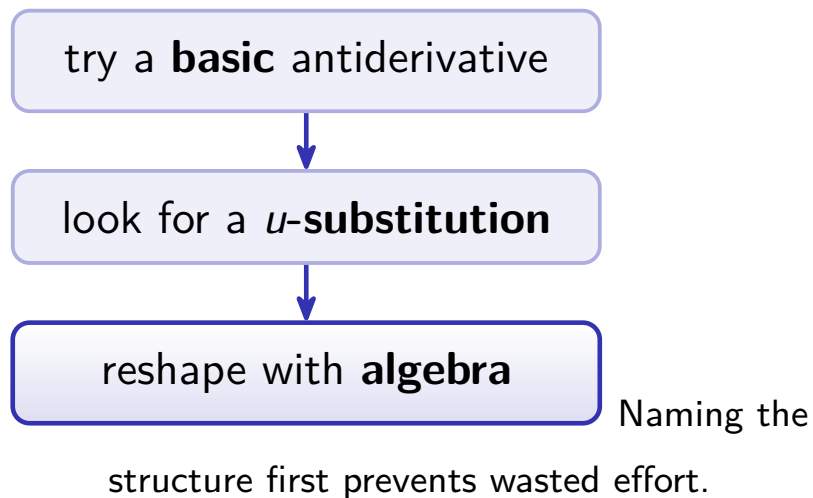
A skill topic: match the integral to a method.

Try a basic antiderivative first; look for a u -substitution (an inside function whose derivative is also present); use algebra (division, completing the square).

Reshaping an integrand, and choosing a technique

Two algebraic set-up moves:

- **long division** when the top degree $\geq$ the bottom
- **completing the square** to reach an arctangent or log form



Exam tips

- Integration is antidifferentiation; use the **power rule** $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ and don't forget the $+C$.
- The **Fundamental Theorem** links the two: $\int_a^b f'(x) dx = f(b) - f(a)$, and $\frac{d}{dx} \int_a^x f(t) dt = f(x)$.
- Approximate a definite integral with **Riemann sums** or the trapezoidal rule from a table of values.
- A definite integral is a signed area (below the axis counts negative); split at sign changes for total area.
- Use **u-substitution** and remember to change the limits (or back-substitute) accordingly.

Exercises 2.1-2.3

2.1 Find $\int x^5 dx$.

[1]

2.2 Find $\int (4x^3 - 2x) dx$.

[2]

2.3 Find $\int \left(\frac{1}{x} + e^x \right) dx$.

[2]

Exercises 2.1-2.3

2.1 Find $\int x^5 dx$.

[1]

2.1

- $\frac{x^6}{6} + C$

2.2 Find $\int (4x^3 - 2x) dx$.

[2]

2.2

- $x^4 - x^2 + C$

2.3 Find $\int \left(\frac{1}{x} + e^x \right) dx$.

[2]

2.3

- $\ln |x| + e^x + C$

Exercise 3.1

3.1 $\int \cos x \, dx =$

[1]

☐**A** $-\sin x + C$ **B** $\sin x + C$ **C** $-\cos x + C$ **D** $\tan x + C$

Exercise 3.1

3.1 $\int \cos x \, dx =$

[1]

☐

A $-\sin x + C$

B $\sin x + C$

C $-\cos x + C$

D $\tan x + C$

3.1 B

- $\int \cos x \, dx = \sin x + C$

Exercises 3.2-3.3

3.2 (a) Find the general antiderivative of $f(x) = 3x^2 + 4x$. [2]

(b) Find the particular antiderivative F with $F(0) = 5$. [2]

3.3 Find $\int \left(\sqrt{x} + \frac{2}{x} \right) dx$. Write $\sqrt{x}$ as $x^{1/2}$ first. [3]

Exercises 3.2-3.3

3.2 (a) Find the general antiderivative of $f(x) = 3x^2 + 4x$. [2]

(b) Find the particular antiderivative F with $F(0) = 5$. [2]

3.2

(a)

- $F(x) = x^3 + 2x^2 + C$

(b)

- $F(0) = C = 5$, so $F(x) = x^3 + 2x^2 + 5$

3.3 Find $\int \left(\sqrt{x} + \frac{2}{x} \right) dx$. Write $\sqrt{x}$ as $x^{1/2}$ first. [3]

3.3

- $\int (x^{1/2} + 2x^{-1}) dx = \frac{x^{3/2}}{3/2} + 2 \ln |x| + C = \frac{2}{3}x^{3/2} + 2 \ln |x| + C$