

Exercise walk-through

6.7

The Fundamental Theorem of Calculus and Definite Integrals

AP Calculus AB

Accumulation functions and the FTC

A definite integral with a **variable upper limit** is an **accumulation function**. The

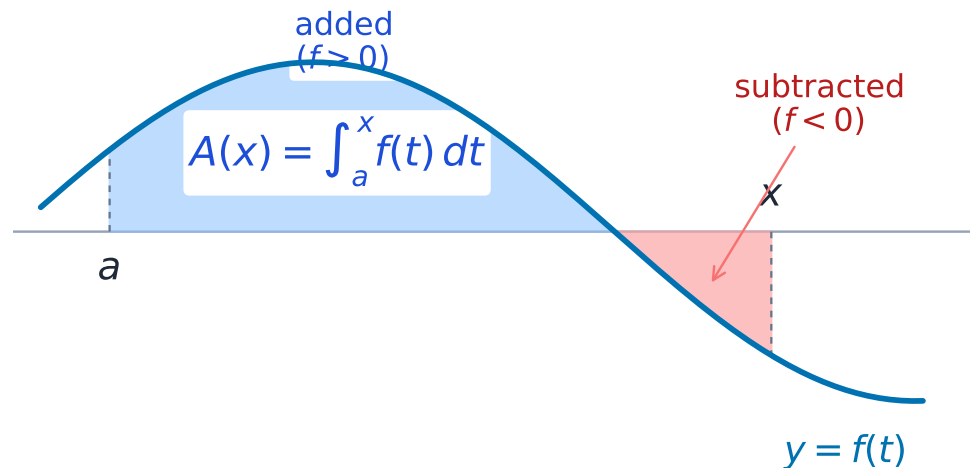
Fundamental Theorem

says differentiation undoes it:

$$\frac{d}{dx} \int_a^x f(t) dt = f(x).$$

So $g(x) = \int_a^x f$ has $g' = f$ and $g'' = f'$.

$$A'(x) = f(x) \quad (\text{Fundamental Theorem})$$



Behaviour of an accumulation function

Because $g'(x) = f(x)$, everything from Unit 5 applies – read it off the graph of f :

g **increasing** where $f > 0$,
decreasing where $f < 0$

g has a **local extremum**
where f **crosses zero**

g **concave up** where f is **increasing**

a **value** of g is the
signed area up to x

Properties of definite integrals

$$\int_a^a f = 0$$

$$\int_b^a f = - \int_a^b f$$

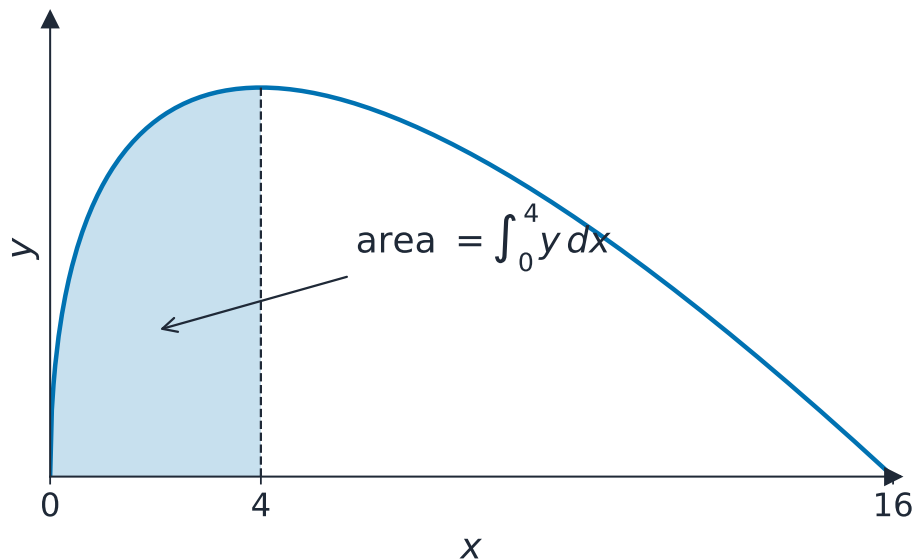
$$\int_a^b kf = k \int_a^b f$$

$$\int_a^b (f \pm g) = \int_a^b f \pm \int_a^b g$$

$$\int_a^c f + \int_c^b f = \int_a^b f$$

The last one – **splitting at an interior point** – lets you build a total from pieces read off a graph.

The Fundamental Theorem and Evaluating Integrals



A definite integral is the signed area between the curve and the x-axis

The theorem, and one algebraic tool

Part 1

The **fundamental theorem of calculus**: if $F(x) = \int_a^x f(t) dt$ then $F'(x) = f(x)$ – differentiation undoes accumulation.

Part 2

$\int_a^b f(x) dx = F(b) - F(a)$ for any antiderivative F – which is what makes a definite integral computable.

With a chain rule

If the upper limit is $g(x)$, then the derivative is $f(g(x)) g'(x)$.

Polynomial long division

Polynomial long division first when the integrand is an improper rational function – the quotient integrates directly, and only the remainder needs more work.

The two parts say the same thing twice: accumulation and rate are inverse operations.

Exercises 2.1·2.2

2.1 Evaluate $\int_0^4 x \, dx$. [2]

2.2 Evaluate $\int_1^2 (4x^3) \, dx$. [2]

Exercises 2.1·2.2

2.1 Evaluate $\int_0^4 x \, dx$.

[2]

2.1

- $\left[\frac{1}{2}x^2\right]_0^4 = 8 - 0 = 8$

2.2 Evaluate $\int_1^2 (4x^3) \, dx$.

[2]

2.2

- $\left[x^4\right]_1^2 = 16 - 1 = 15$

Exercises 2.3-3.1

2.3 Evaluate $\int_0^{\pi/2} \cos x \, dx$.

[2]

3.1 $\int_a^b f'(x) \, dx =$

[1]

☐

A $f'(b) - f'(a)$

B $f(b) - f(a)$

C $f(b) + f(a)$

D $\frac{1}{2}(f(a) + f(b))$

Exercises 2.3-3.1

2.3 Evaluate $\int_0^{\pi/2} \cos x \, dx$.

[2]

2.3

- $\left[\sin x \right]_0^{\pi/2} = 1 - 0 = 1$

3.1 $\int_a^b f'(x) \, dx =$

[1]

☐

A $f'(b) - f'(a)$

B $f(b) - f(a)$

C $f(b) + f(a)$

D $\frac{1}{2}(f(a) + f(b))$

3.1 B

- by the FTC, $\int_a^b f' = f(b) - f(a)$

Exercise 3.2 (a)

3.2 A particle's velocity is $v(t) = 3t^2 - 4$ m/s.

(a) Evaluate $\int_0^{\quad} v(t) \, dt$.

[2]

Exercise 3.2 (a)

3.2 A particle's velocity is $v(t) = 3t^2 - 4$ m/s.

(a) Evaluate $\int_0^8 v(t) dt$.

[2]

(a)

- $[t^3 - 4t]_0^8 = (8 - 8) - 0 = 0$

Exercise 3.2 (b)

3.2 A particle's velocity is $v(t) = 3t^2 - 4$ m/s.

(b) State what this value represents physically.

[1]

Exercise 3.2 (b)

3.2 A particle's velocity is $v(t) = 3t^2 - 4$ m/s.

(b) State what this value represents physically.

[1]

(b)

- The particle's **displacement** over $0 \leq t \leq 2$ is 0 – it returns to its start

Exercise 3.3

3.3 Water enters a tank at rate $r(t) = 6t$ L/min. Find the total volume added between $t = 1$ and $t = 4$ minutes. [3]

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3.3

- $\int_1^4 6t \, dt = [3t^2]_1^4 = 48 - 3 = 45 \text{ L}$