Exercise walk-through

6.6
Applying Properties of Definite Integrals

AP Calculus AB

Finding an area without an antiderivative, and its units

geometry	simple regions – triangles and rectangles – can be found by area formula alone
a left or right sum	rectangles at one end of each subinterval
a midpoint sum	rectangles at the centre – usually the most accurate of the three
a trapezoidal sum	the average of the left and right sums; it overestimates a concave-up function
the units	the area's unit is the rate's unit times the input's unit
the example	a rate in vehicles per hour, integrated over hours, gives vehicles

> Stating the units of a definite integral is a marked step on the free-response paper, and it is also the fastest check that you integrated the right thing.

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The theorem, and one algebraic tool

Part 1	The **fundamental theorem of calculus**: if $F(x) = \int_a^x f(t)\,dt$ then $F'(x) = f(x)$ – differentiation undoes accumulation.
Part 2	$\int_a^b f(x)\,dx = F(b) - F(a)$ for any antiderivative F – which is what makes a definite integral computable.
With a chain rule	If the upper limit is $g(x)$, then the derivative is $f(g(x))\,g'(x)$.
Polynomial long division	**Polynomial long division** first when the integrand is an improper rational function – the quotient integrates directly, and only the remainder needs more work.

> The two parts say the same thing twice: accumulation and rate are inverse operations.

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Exam tips

- Integration is antidifferentiation; use the **power rule** $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ and don't forget the $+C$.
- The **Fundamental Theorem** links the two: $\int_a^b f'(x)\,dx = f(b) - f(a)$, and $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.
- Approximate a definite integral with **Riemann sums** or the trapezoidal rule from a table of values.
- A definite integral is a signed area (below the axis counts negative); split at sign changes for total area.
- Use **u-substitution** and remember to change the limits (or back-substitute) accordingly.

Exercises 2.1·2.2

2.1 Find $\int_0^7 f\,dx$. [1]

2.2 Find $\int_3^0 f\,dx$. [1]

Exercises 2.1·2.2

2.1 Find $\displaystyle\int_0^7 f\,dx$. [1]

2.1

- $\int_0^7 f = \int_0^3 f + \int_3^7 f = 6 + (-2) = 4$

2.2 Find $\displaystyle\int_3^0 f\,dx$. [1]

2.2

- $\int_3^0 f = -\int_0^3 f = -6$

Exercises 2.3·3.1

2.3 Find $\displaystyle\int_0^3 (2f + g)\,dx$. [2]

3.1 $\displaystyle\int_5 f(x)\,dx =$ [1]

- **A** $f(5)$
- **B** 1
- **C** 0
- **D** undefined

Exercises 2.3·3.1

2.3 Find $\int_0^3 (2f + g)\,dx$. [2]

2.3

- $2(6) + 5 = 17$

3.1 $\int_5 f(x)\,dx =$ [1]

- [ ] **A** $f(5)$
- **B** 1
- **C** 0
- **D** undefined

3.1 C

- a zero-width interval gives 0

Exercise 3.2 (a)

3.2 You are given $\int_1^6 h = 12$ and $\int_1^4 h = 7$.

(a) Find $\int_4^{\breve{}} h\,dx$. [2]

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(a) Find $\int_4^{\breve{}} h\,dx$. [2]

(a)

- $\int_4^6 h = \int_1^6 h - \int_1^4 h = 12 - 7 = 5$

Exercise 3.2 (b)

3.2 You are given $\int_1^6 h = 12$ and $\int_1^4 h = 7$.

(b) Find $\int_6^{\hat{}} h\,dx$. [1]

Exercise 3.2 (b)

3.2 You are given $\int_1^6 h = 12$ and $\int_1^4 h = 7$.

(b) Find $\int_6^{\hat{}} h\,dx$. [1]

(b)

- $\int_6^1 h = -12$

Exercise 3.3

3.3 Given $\int_0^2 p = 8$ and $\int_0^2 q = -3$, find $\int_0^2 (5p - 4q)\,dx$. [2]

$\int_0^2 p = 8$ $\int_0^2 q = -3$

Exercise 3.3

3.3 Given $\int_0^2 p = 8$ and $\int_0^2 q = -3$, find $\int_0^2 (5p - 4q)\,dx$. [2]

$\int_0^2 p = 8$ $\int_0^2 q = -3$

3.3

- $5(8) - 4(-3) = 40 + 12 = 52$