Exercise walk-through

6.5 Interpreting the Behavior of Accumulation Functions Involving Area

AP Calculus AB

Accumulation of change

Differentiation found rates. Integration runs it in reverse: given a rate, find the **accumulated change** .

> The area between a rate graph and the x-axis is the total accumulation.

Water tank

Exercises 2.1·2.3

2.1 On an interval where $f(t) < 0$, state whether g is increasing or decreasing. [1]

2.2 At a point where f changes from positive to negative, what feature does g have? [1]

2.3 On an interval where f is increasing, state the concavity of g. [1]

Exercises 2.1·2.3

2.1 On an interval where $f(t) < 0$, state whether g is increasing or decreasing. [1]

2.1

- Decreasing – $g'(x) = f(x) < 0$

2.2 At a point where f changes from positive to negative, what feature does g have? [1]

2.2

- A local maximum

2.3 On an interval where f is increasing, state the concavity of g. [1]

2.3

- Concave up – $g''(x) = f'(x) > 0$

Exercise 3.1

3.1 $g(x) = \int_0^x f(t)\,dt$. The graph of f crosses from below the axis to above it at $x = 4$. At $x = 4$, g has a [1]

- **A** local maximum
- **B** local minimum
- **C** point of inflection
- **D** vertical asymptote

Exercise 3.1

3.1 $g(x) = \int_0^x f(t)\,dt$. The graph of f crosses from below the axis to above it at $x = 4$. At $x = 4$, g has a [1]

- ☐ **A** local maximum
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3.1 B

- f changes $-$ to $+$, so g' changes sign the same way and g has a local minimum

Exercise 3.2 (a)

3.2 The graph of f consists of straight segments: $f > 0$ on $(0, 3)$, $f < 0$ on $(3, 7)$, and $f(3) = 0$. Let $g(x) = \int_0^x f(t)\,dt$.

(a) On what interval is g increasing? [1]

Exercise 3.2 (a)

3.2 The graph of f consists of straight segments: $f > 0$ on $(0, 3)$, $f < 0$ on $(3, 7)$, and $f(3) = 0$. Let $g(x) = \int_0^x f(t)\,dt$.

(a) On what interval is g increasing? [1]

(a)

- g increases on $(0, 3)$ where $f > 0$

Exercise 3.2 (b)

3.2 The graph of f consists of straight segments: $f > 0$ on $(0, 3)$, $f < 0$ on $(3, 7)$, and $f(3) = 0$. Let $g(x) = \int_0^x f(t)\,dt$.

(b) At what x does g attain a local maximum? Justify. [2]

Exercise 3.2 (b)

3.2 The graph of f consists of straight segments: $f > 0$ on $(0, 3)$, $f < 0$ on $(3, 7)$, and $f(3) = 0$. Let $g(x) = \int_0^x f(t)\,dt$.

(b) At what x does g attain a local maximum? Justify. [2]

(b)

- At $x = 3$, because f (which is g') changes from positive to negative there

Exercise 3.3

3.3 For $g(x) = \int_0^x f(t)\,dt$, the graph of f is increasing on $(1, 5)$. State, with a reason, the concavity of g on $(1, 5)$. [2]

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3.3 For $g(x) = \int_0^x f(t)\,dt$, the graph of f is increasing on $(1, 5)$. State, with a reason, the concavity of g on $(1, 5)$. [2]

3.3

- Concave up – $g''(x) = f'(x) > 0$ since f is increasing