

Exercise walk-through

6.2

Approximating Areas with Riemann Sums

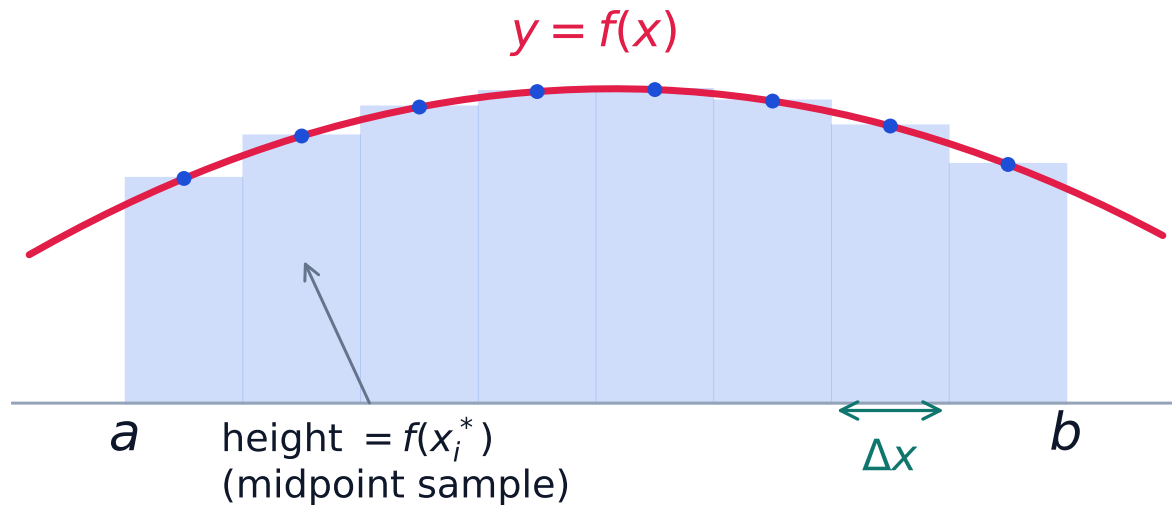
AP Calculus AB

Approximating area with a Riemann sum

Split the interval and add rectangle areas – a **Riemann sum**. Each rectangle is $f(x) \Delta x$.

As the strips narrow, the sum approaches the [definite integral](#).

Approximating area with a Riemann sum, in detail



$$\sum f(x_i^*) \Delta x \rightarrow \int_a^b f \text{ as } n \rightarrow \infty$$

Split the interval and add rectangle areas – a **Riemann sum** .

The four standard estimates

Left sum – height from
each strip's **left** endpoint

Right sum – height
from the **right** endpoint

Midpoint sum – height
from the **midpoint**

Trapezoidal sum –
average the two endpoint heights

Strips may be **uniform** or **nonuniform** – read the widths from the table.

Over- or underestimate?

Judge from the **behaviour** of the function:

- **increasing** f : left sum **under**, right sum **over**
- **concave up**: trapezoidal **over**
- **concave down**: trapezoidal **under**

Finding an area without an antiderivative, and its units

geometry

simple regions – triangles and rectangles – can be found by area formula alone

a left or right sum

rectangles at one end of each subinterval

a midpoint sum

rectangles at the centre – usually the most accurate of the three

a trapezoidal sum

the average of the left and right sums; it overestimates a concave-up function

the units

the area's unit is the rate's unit times the input's unit

the example

a rate in vehicles per hour, integrated over hours, gives vehicles

Stating the units of a definite integral is a marked step on the free-response paper, and it is also the fastest check that you integrated the right thing.

Exercises 2.1-2.3

2.1 Find the **left** Riemann sum for $\int_0^9 g \, dx$ with the three subintervals shown. [2]

2.2 Find the **right** Riemann sum for the same integral. [2]

2.3 Find the **trapezoidal** estimate. [2]

Exercises 2.1-2.3

2.1 Find the **left** Riemann sum for $\int_0^9 g \, dx$ with the three subintervals shown. [2]

2.1

- $\Delta x = 3$; left $= 3(2 + 5 + 6) = 39$

2.2 Find the **right** Riemann sum for the same integral. [2]

2.2

- right $= 3(5 + 6 + 10) = 63$

2.3 Find the **trapezoidal** estimate. [2]

2.3

- $3\left[\frac{2}{2} + 5 + 6 + \frac{10}{2}\right] = 3(17) = 51$. (Equivalently the average of 39 and 63.)

Exercise 3.1

3.1 For an increasing function, a **left** Riemann sum gives an

[1]

☐

A over-estimate

B under-estimate

C exact value

D value that depends on concavity

Exercise 3.1

3.1 For an increasing function, a **left** Riemann sum gives an

[1]

☐

A over-estimate

B under-estimate

C exact value

D value that depends on concavity

3.1 B

- for an increasing function the left sum under-estimates

Exercise 3.2 (a)

3.2 A car's speed $v(t)$ in m/s is recorded:

t (s)	0	4	8	12
$v(t)$	0	12	20	24

(a) Estimate the distance travelled over $0 \leq t \leq 12$ using a **right** Riemann sum with three subintervals. [2]

Exercise 3.2 (a)

3.2 A car's speed $v(t)$ in m/s is recorded:

t (s)	0	4	8	12
$v(t)$	0	12	20	24

(a) Estimate the distance travelled over $0 \leq t \leq 12$ using a **right** Riemann sum with three subintervals. [2]

(a)

- $4(12 + 20 + 24) = 224 \text{ m}$

Exercise 3.2 (b)

3.2 A car's speed $v(t)$ in m/s is recorded:

t (s)	0	4	8	12
$v(t)$	0	12	20	24

(b) Is this an over- or under-estimate of the true distance? Give a reason.

[1]

Exercise 3.2 (b)

3.2 A car's speed $v(t)$ in m/s is recorded:

t (s)	0	4	8	12
$v(t)$	0	12	20	24

(b) Is this an over- or under-estimate of the true distance? Give a reason. [1]

(b)

- Over-estimate – v is increasing, so right endpoints are the largest values on each subinterval

Exercise 3.3

3.3 The function f is concave up on $[0, 4]$. A student estimates $\int_0^4 f \, dx$ with a trapezoidal sum. State, with a reason, whether the estimate is too large or too small. [2]

Exercise 3.3

3.3 The function f is concave up on $[0, 4]$. A student estimates $\int_0^4 f \, dx$ with a trapezoidal sum. State, with a reason, whether the estimate is too large or too small. [2]

3.3

- Too large – for a concave-up curve each trapezoid lies **above** the curve, so the trapezoidal sum over-estimates