Exercise walk-through

6.14 Selecting Techniques for Antidifferentiation

AP Calculus AB

Finding an area without an antiderivative, and its units

geometry	simple regions – triangles and rectangles – can be found by area formula alone
a left or right sum	rectangles at one end of each subinterval
a midpoint sum	rectangles at the centre – usually the most accurate of the three
a trapezoidal sum	the average of the left and right sums; it overestimates a concave-up function
the units	the area's unit is the rate's unit times the input's unit
the example	a rate in vehicles per hour, integrated over hours, gives vehicles

> Stating the units of a definite integral is a marked step on the free-response paper, and it is also the fastest check that you integrated the right thing.

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The theorem, and one algebraic tool

Part 1	The **fundamental theorem of calculus**: if $F(x) = \int_a^x f(t)\,dt$ then $F'(x) = f(x)$ – differentiation undoes accumulation.
Part 2	$\int_a^b f(x)\,dx = F(b) - F(a)$ for any antiderivative F – which is what makes a definite integral computable.
With a chain rule	If the upper limit is $g(x)$, then the derivative is $f(g(x))\,g'(x)$.
Polynomial long division	**Polynomial long division** first when the integrand is an improper rational function – the quotient integrates directly, and only the remainder needs more work.

> The two parts say the same thing twice: accumulation and rate are inverse operations.

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Exam tips

- Integration is antidifferentiation; use the **power rule** $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ and don't forget the $+C$.
- The **Fundamental Theorem** links the two: $\int_a^b f'(x)\,dx = f(b) - f(a)$, and $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.
- Approximate a definite integral with **Riemann sums** or the trapezoidal rule from a table of values.
- A definite integral is a signed area (below the axis counts negative); split at sign changes for total area.
- Use **u-substitution** and remember to change the limits (or back-substitute) accordingly.

Exercises 2.1–2.3

2.1 $\displaystyle\int \frac{1}{x^2+4}\,dx$ [1]

2.2 $\displaystyle\int 5x^4(x^5+2)^7\,dx$ [1]

2.3 $\displaystyle\int \frac{x^2+1}{x}\,dx$ [1]

Exercises 2.1·2.3

2.1 $\displaystyle\int \frac{1}{x^2+4}\,dx$ [1]

2.1

- Complete-the-square / arctangent form (basic arctangent rule)

2.2 $\displaystyle\int 5x^4(x^5+2)^7\,dx$ [1]

2.2

- Substitution, $u = x^5 + 2$

2.3 $\displaystyle\int \frac{x^2+1}{x}\,dx$ [1]

2.3

- Basic – split into $x + \frac{1}{x}$ first

Exercises 3.1·3.2

3.1 Which integral is best done by **substitution**? [1]

- ☐ **A** $\displaystyle\int (x^3 - 2)\,dx$
- **B** $\displaystyle\int \frac{1}{x^2 + 9}\,dx$
- **C** $\displaystyle\int \cos(x^2) \cdot 2x\,dx$
- **D** $\displaystyle\int \frac{x^2}{x + 1}\,dx$

3.2 Find $\displaystyle\int \frac{x^2 + 3}{x}\,dx$ (rewrite the fraction first). [3]

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3.1 C

- $2x$ is the derivative of x^2, so substitute $u = x^2$

3.2 Find $\displaystyle\int \frac{x^2 + 3}{x}\,dx$ (rewrite the fraction first). [3]

3.2

- $\displaystyle\frac{x^2 + 3}{x} = x + \frac{3}{x}$; $\int \left(x + \frac{3}{x}\right)dx = \frac{x^2}{2} + 3\ln|x| + C$

Exercise 3.3

3.3 Find $\displaystyle\int \frac{6x}{x^2+1}\,dx.$ [3]

$$f(x) = \frac{\boxed{6x}\ \text{numerator}}{\boxed{x^2+1}\ \text{denominator}}$$

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$f(x) =$ $\dfrac{\boxed{6x}\ \text{numerator}}{\boxed{x^2+1}\ \text{denominator}}$

3.3

- $u = x^2+1,\ du = 2x\,dx,$ so $6x\,dx = 3\,du;\ \int \frac{3}{u}\,du = 3\ln|u| = 3\ln(x^2+1) + C$