

Exercise walk-through

6.10

Integrating Functions Using Long Division and Completing the Square

AP Calculus AB

The theorem, and one algebraic tool

Part 1

The **fundamental theorem of calculus**: if $F(x) = \int_a^x f(t) dt$ then $F'(x) = f(x)$ – differentiation undoes accumulation.

Part 2

$\int_a^b f(x) dx = F(b) - F(a)$ for any antiderivative F – which is what makes a definite integral computable.

With a chain rule

If the upper limit is $g(x)$, then the derivative is $f(g(x)) g'(x)$.

Polynomial long division

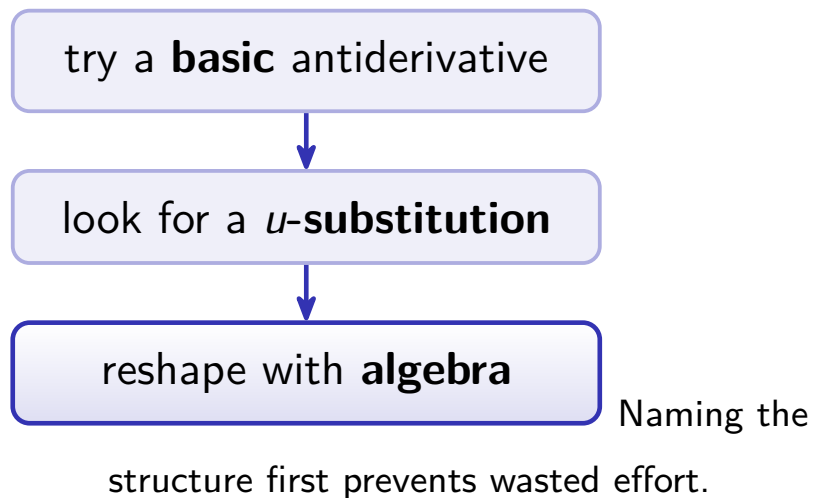
Polynomial long division first when the integrand is an improper rational function – the quotient integrates directly, and only the remainder needs more work.

The two parts say the same thing twice: accumulation and rate are inverse operations.

Reshaping an integrand, and choosing a technique

Two algebraic set-up moves:

- **long division** when the top degree $\geq$ the bottom
- **completing the square** to reach an arctangent or log form



Exam tips

- Integration is antidifferentiation; use the **power rule** $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ and don't forget the $+C$.
- The **Fundamental Theorem** links the two: $\int_a^b f'(x) dx = f(b) - f(a)$, and $\frac{d}{dx} \int_a^x f(t) dt = f(x)$.
- Approximate a definite integral with **Riemann sums** or the trapezoidal rule from a table of values.
- A definite integral is a signed area (below the axis counts negative); split at sign changes for total area.
- Use **u-substitution** and remember to change the limits (or back-substitute) accordingly.

Exercises 2.1·2.3

2.1 Use long division to rewrite $\frac{x+3}{x+1}$ as $1 + \frac{?}{x+1}$. [2]

2.2 Find $\int \frac{1}{x^2+16} dx$. [2]

2.3 Complete the square: write $x^2 + 6x + 13$ in the form $(x+a)^2 + b$. [2]

Exercises 2.1-2.3

2.1 Use long division to rewrite $\frac{x+3}{x+1}$ as $1 + \frac{?}{x+1}$. [2]

2.1

- $\frac{x+3}{x+1} = 1 + \frac{2}{x+1}$

2.2 Find $\int \frac{1}{x^2+16} dx$. [2]

2.2

- $\frac{1}{4} \arctan \frac{x}{4} + C$

2.3 Complete the square: write $x^2 + 6x + 13$ in the form $(x+a)^2 + b$. [2]

2.3

- $(x+3)^2 + 4$

Exercises 3.1·3.2

3.1 $\int \frac{1}{x^2 + 25} dx =$

[1]

☐

A $\arctan(x) + C$

B $\frac{1}{5} \arctan \frac{x}{5} + C$

C $\ln(x^2 + 25) + C$

D $5 \arctan(5x) + C$

3.2 Find $\int \frac{x+2}{x+1} dx$.

[3]

Exercises 3.1·3.2

3.1 $\int \frac{1}{x^2 + 25} dx =$

[1]



A $\arctan(x) + C$

B $\frac{1}{5} \arctan \frac{x}{5} + C$

C $\ln(x^2 + 25) + C$

D $5 \arctan(5x) + C$

3.1 B

- with $a = 5$, $\int \frac{dx}{x^2+25} = \frac{1}{5} \arctan \frac{x}{5} + C$

3.2 Find $\int \frac{x+2}{x+1} dx$.

[3]

3.2

- $\frac{x+2}{x+1} = 1 + \frac{1}{x+1}$; $\int \left(1 + \frac{1}{x+1}\right) dx = x + \ln|x+1| + C$

Exercise 3.3

3.3 Find $\int \frac{1}{x^2 + 2x + 10} dx$ by completing the square.

[4]

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3.3

- $x^2 + 2x + 10 = (x + 1)^2 + 9$; with $u = x + 1$, $\int \frac{du}{u^2 + 9} = \frac{1}{3} \arctan \frac{u}{3} = \frac{1}{3} \arctan \frac{x + 1}{3} + C$