Exercise walk-through

6.1 Exploring Accumulations of Change

AP Calculus AB

You must be able to

- interpret the **area** under a rate-of-change graph as an **accumulated change**
- treat area **above** the axis as positive and area **below** as negative
- attach correct **units** (rate unit $\times$ time unit) to an accumulation

1 The method

Method — Area under a rate graph is a total

If water flows in at a **rate** $R(t)$ litres per minute, the total water added between $t = 0$ and $t = 5$ is the **area** under the R-vs-t graph over $[0, 5]$. A rate graph that is a rectangle of height 3 L/min over 5 min accumulates $3 \times 5 = 15$ L.

Exercises 2.1–2.3

2.1 A tap runs at a constant 2.5 L/min for 8 minutes. How much water flows out? [1]

2.2 A rate graph is a triangle: the rate rises linearly from 0 to 10 units/s over 4 s. Find the accumulated change. [2]

2.3 Over one interval a quantity gains an area of 12 and over the next it loses an area of 5. State the net accumulated change. [1]

Exercises 2.1–2.3

2.1 A tap runs at a constant 2.5 L/min for 8 minutes. How much water flows out? [1]

2.1

- $2.5 \times 8 = 20$ L

2.2 A rate graph is a triangle: the rate rises linearly from 0 to 10 units/s over 4 s. Find the accumulated change. [2]

2.2

- Triangle area $= \frac{1}{2}(4)(10) = 20$ units

2.3 Over one interval a quantity gains an area of 12 and over the next it loses an area of 5. State the net accumulated change. [1]

2.3

- $12 - 5 = 7$

Exercise 3.1

3.1 The area under a velocity-vs-time graph represents [1]

- [ ] **A** acceleration
- [ ] **B** displacement
- [ ] **C** speed
- [ ] **D** force

Exercise 3.1

3.1 The area under a velocity-vs-time graph represents [1]

- ☐ **A** acceleration
- **B** displacement
- **C** speed
- **D** force

3.1 B

- the area under a velocity–time graph is displacement

Exercise 3.2 (a)

3.2 A machine consumes power at a rate $P(t)$ kilowatts, shown as a graph made of straight segments: a rectangle of height 2 over $0 \le t \le 3$ hours, then a triangle rising from 0 to 4 over $3 \le t \le 5$ hours.

(a) Find the energy used over $0 \le t \le 3$. [1]

Exercise 3.2 (a)

3.2 A machine consumes power at a rate $P(t)$ kilowatts, shown as a graph made of straight segments: a rectangle of height 2 over $0 \le t \le 3$ hours, then a triangle rising from 0 to 4 over $3 \le t \le 5$ hours.

(a) Find the energy used over $0 \le t \le 3$. [1]

(a)

- $2 \times 3 = 6$ kWh

Exercise 3.2 (b)

3.2 A machine consumes power at a rate $P(t)$ kilowatts, shown as a graph made of straight segments: a rectangle of height 2 over $0 \le t \le 3$ hours, then a triangle rising from 0 to 4 over $3 \le t \le 5$ hours.

(b) Find the energy used over $3 \le t \le 5$. [2]

Exercise 3.2 (b)

3.2 A machine consumes power at a rate $P(t)$ kilowatts, shown as a graph made of straight segments: a rectangle of height 2 over $0 \le t \le 3$ hours, then a triangle rising from 0 to 4 over $3 \le t \le 5$ hours.

(b) Find the energy used over $3 \le t \le 5$. [2]

(b)

- triangle $\frac{1}{2}(2)(4) = 4$ kWh

Exercise 3.2 (c)

3.2 A machine consumes power at a rate $P(t)$ kilowatts, shown as a graph made of straight segments: a rectangle of height 2 over $0 \le t \le 3$ hours, then a triangle rising from 0 to 4 over $3 \le t \le 5$ hours.

(c) State the total energy used, with units. [1]

Exercise 3.2 (c)

3.2 A machine consumes power at a rate $P(t)$ kilowatts, shown as a graph made of straight segments: a rectangle of height 2 over $0 \le t \le 3$ hours, then a triangle rising from 0 to 4 over $3 \le t \le 5$ hours.

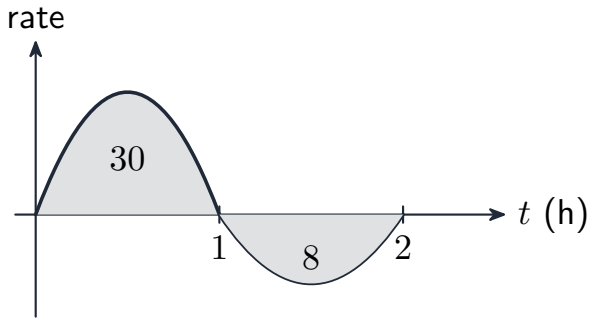
(c) State the total energy used, with units. [1]

(c)

- $6 + 4 = 10$ kWh

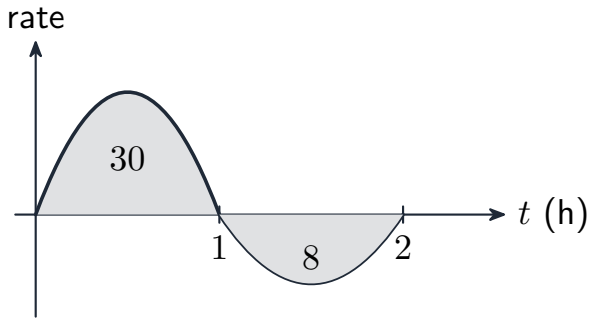
Exercise 3.3

3.3 Sand is added to a pile at a rate that gives an accumulated area of 30 (kg) over the first hour, then removed at a rate giving an area of 8 (kg) over the next hour. State the net change in the pile's mass. [1]



Exercise 3.3

3.3 Sand is added to a pile at a rate that gives an accumulated area of 30 (kg) over the first hour, then removed at a rate giving an area of 8 (kg) over the next hour. State the net change in the pile's mass. [1]



3.3

- $30 - 8 = 22$ kg