

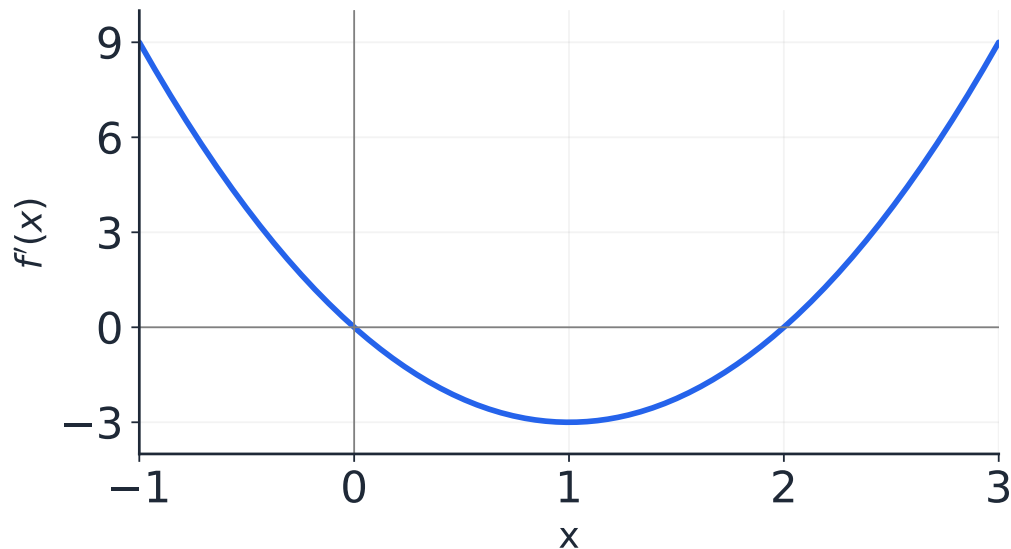
Exercise walk-through

5.9

Connecting a Function, Its First Derivative, and Its Second Derivative

AP Calculus AB

Read a derivative graph to describe the function



Your turn

The graph is $f'(x) = 3x^2 - 6x$ on $(-1, 3)$. Where is f increasing or decreasing? Classify the critical points and justify each answer.

The vertical coordinate is f' , not f . Use the sign on each interval.

Use signs, then name the turning points

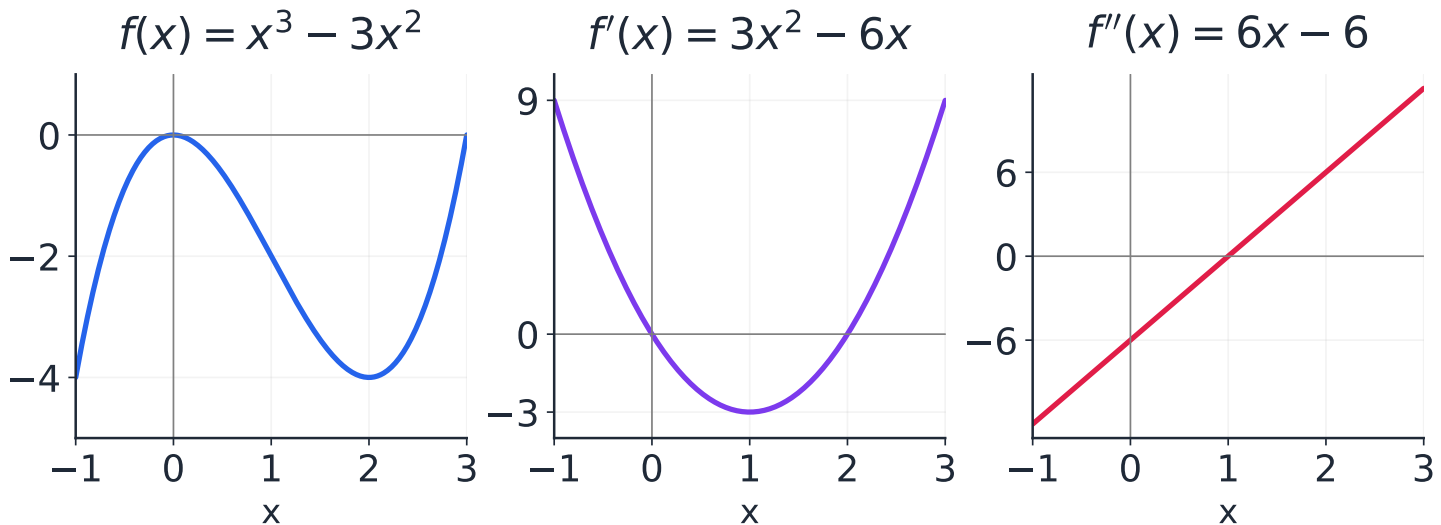
$$f'(x) = 3x(x - 2) = 0 \implies x = 0, 2.$$

Interval	$(-1, 0)$	$(0, 2)$	$(2, 3)$
Sign of f'	+	-	+
Behaviour of f	increasing	decreasing	increasing

State the evidence

At 0, f' changes from positive to negative: a local maximum. At 2, it changes from negative to positive: a local minimum. Zeros alone would only identify candidates.

Match all three graphs to the same cubic



For $f(x) = x^3 - 3x^2$, the extrema are $(0, 0)$ and $(2, -4)$. The derivative zeros align with these x coordinates; the graphs do not have the same heights.

A derivative graph does not fix the vertical position

$$f(x) = x^3 - 3x^2, \quad F(x) = x^3 - 3x^2 + 5.$$

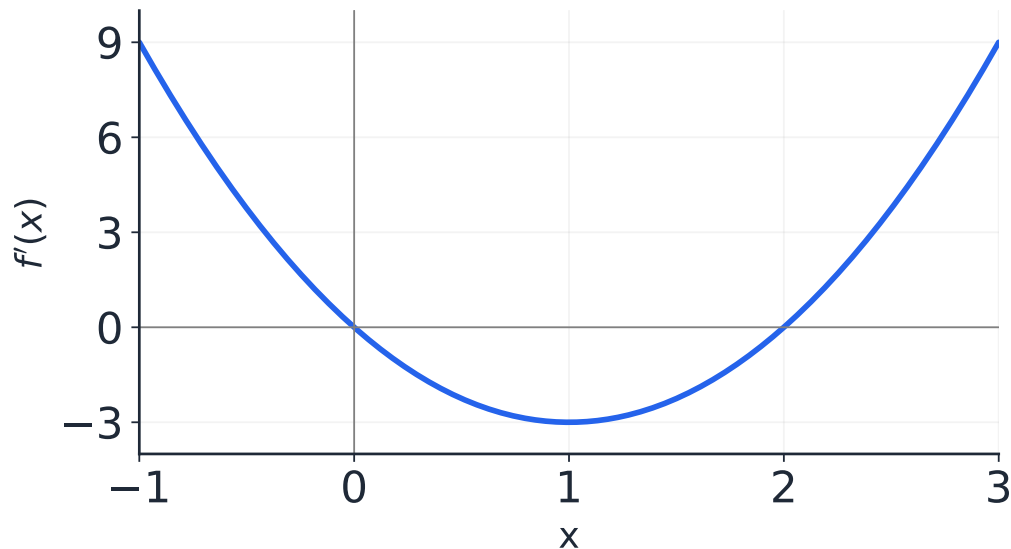
$$F'(x) = f'(x) = 3x^2 - 6x.$$

What stays the same?

Both functions increase and decrease on the same intervals, and their turning points have the same x coordinates. Their y coordinates differ by 5. A value such as $f(0) = 0$ fixes which vertical position is intended.

Sketch the shape from f' ; use a given function value to place it vertically.

Above the axis, but sloping down



Your turn

For $f(x) = x^3 - 3x^2$, decide whether f increases on $(-1, 0)$ and whether it is concave up there. Locate its inflection; justify using the graph of f' .

Height of f' answers one question; increasing or decreasing f' answers another.

Positive derivative can accompany concave down

$$f'(x) = 3x^2 - 6x, \quad f''(x) = 6x - 6.$$

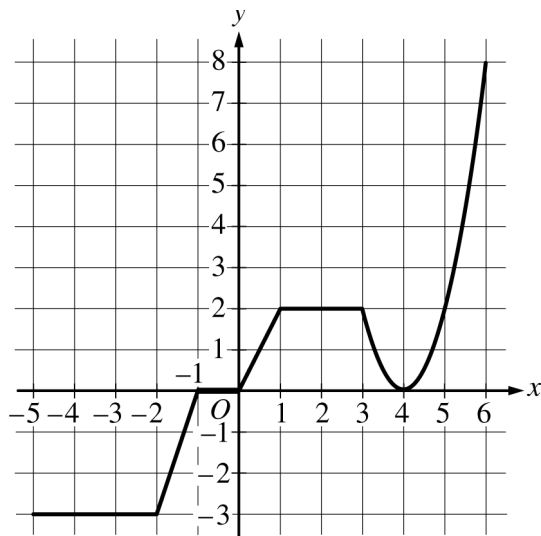
On $(-1, 0)$

$f' > 0$, so f increases. But f' decreases, so f is **concave down**. These conclusions are compatible.

Across $x = 1$

$f' < 0$ for $x < 1$ and $f' > 0$ for $x > 1$. Concavity changes from down to **concave up**, giving a **point of inflection** at $(1, -2)$ for the stated cubic. Here $f'(1) = -3$, not zero.

Original 2018: the graph is g , and g equals f'



Graph of g

2018 AB Q 3(c–d)

Given $g = f'$, find intervals where f is both increasing and concave up. Then find the x coordinate of each inflection point of f . Justify using the given graph.

Original 2018: combine height and direction of change

$$f'(x) = g(x) > 0 \quad \text{and} \quad g \text{ increasing.}$$

Increasing and concave up

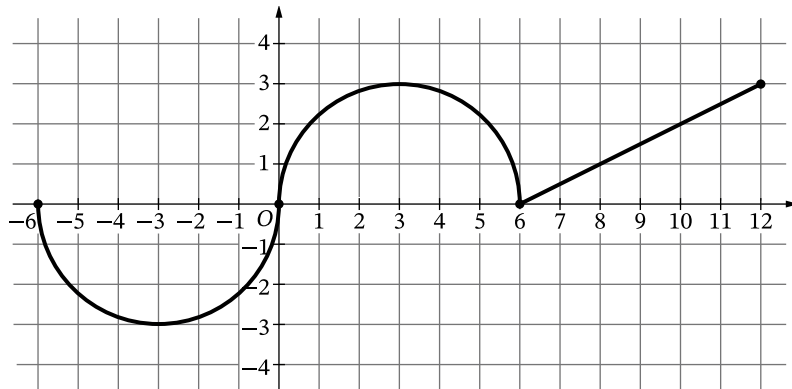
Both hold on $(0, 1)$ and $(4, 6)$. A positive but horizontal graph of g does not establish concave up.

Inflection

At 4, $g = f'$ changes from decreasing to increasing, so f changes concavity. Although $g(4) = 0$, g is positive on both sides: this point is not an extremum of f .

Give both intervals and the graph-based reason.

Original 2025: now the graph is f , and f equals g -prime



Graph of f

2025 AB Q 4B

$g(x) = \int_6^x f(t) dt$, so $g' = f$. Find all inflection x coordinates of g on $(-6, 12)$. Is every zero of the displayed graph an inflection? Give graph-based reasons.

Original 2025: turns of f locate inflections of g

x	Change in $f = g'$
-3	decreasing to increasing
3	increasing to decreasing
6	decreasing to increasing

Why 0 is excluded, and 6 is included

At 0, f increases on both sides, so g stays concave up. At 6, f changes direction although f' is undefined. The neighbouring concavity changes; g'' need not exist at the point itself.

Justify each answer using the increasing/decreasing behaviour of the displayed f .

Exercises 2.1·2.3

2.1 State what f'' tells you about f . [1]

2.2 State sufficient conditions on f' and f'' for the second derivative test to prove a local minimum. [1]

2.3 For $f(x) = x^3$, state where $f' = 0$ and where $f'' = 0$. [2]

Exercises 2.1-2.3

2.1 State what f'' tells you about f . [1]

2.1

- its concavity (concave up where $f'' > 0$, down where $f'' < 0$)
-

2.2 State sufficient conditions on f' and f'' for the second derivative test to prove a local minimum. [1]

2.2

- $f' = 0$ and $f'' > 0$
-

2.3 For $f(x) = x^3$, state where $f' = 0$ and where $f'' = 0$. [2]

2.3

- $f' = 0$ at $x = 0$; $f'' = 0$ at $x = 0$

Exercises 3.1-3.2

3.1 f'' gives information about a graph's

[1]

☐

A slope

B concavity

C x -intercepts

D domain

☐

3.2 Which conditions prove a local minimum of f by the second derivative test?

[1]

A $f' = 0$, $f'' > 0$

B $f' = 0$, $f'' < 0$

C $f'' = 0$ only

D $f = 0$

Exercises 3.1-3.2

3.1 f'' gives information about a graph's

[1]

☐

A slope

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C x -intercepts

D domain

3.1 B

- concavity.

3.2 Which conditions prove a local minimum of f by the second derivative test?

[1]

☐

A $f' = 0$, $f'' > 0$

B $f' = 0$, $f'' < 0$

C $f'' = 0$ only

D $f = 0$

3.2 A

- $f' = 0$, $f'' > 0$.

Exercise 3.3 (a)

3.3 $f(x) = x^3$.

(a) Find $f'(x)$.

[1]

Exercise 3.3 (a)

3.3 $f(x) = x^3$.

(a) Find $f'(x)$.

[1]

(a)

- $3x^2$

Exercise 3.3 (b)

3.3 $f(x) = x^3$.

(b) State where $f' = 0$.

[1]

Exercise 3.3 (b)

3.3 $f(x) = x^3$.

(b) State where $f' = 0$.

[1]

(b)

- $x = 0$

Exercise 3.3 (c)

3.3 $f(x) = x^3$.

(c) State where $f'' = 0$.

[1]

Exercise 3.3 (c)

3.3 $f(x) = x^3$.

(c) State where $f'' = 0$.

[1]

(c)

- $x = 0$