

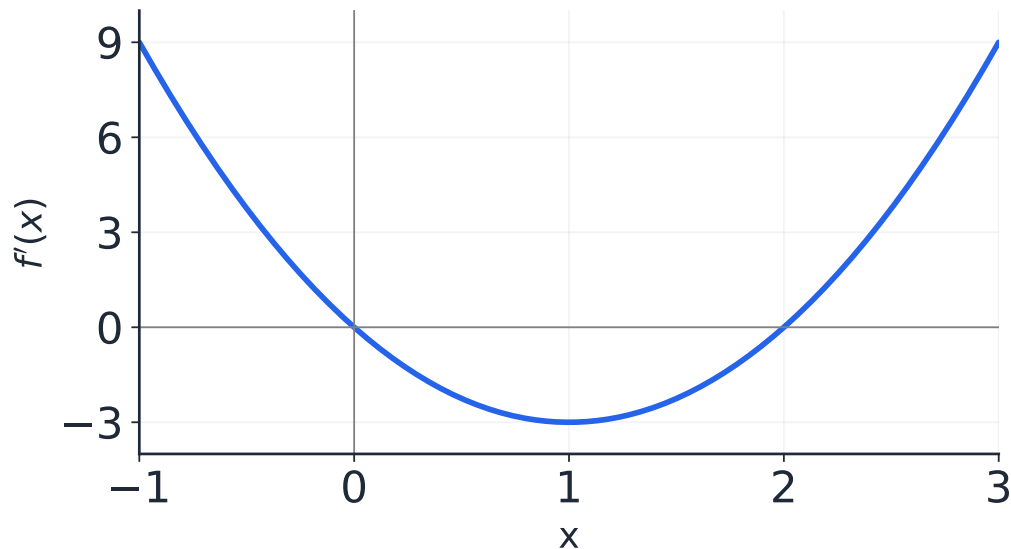
Exercise walk-through

5.6

Determining Concavity of Functions over Their Domains

AP Calculus AB

Above the axis, but sloping down



Your turn

For $f(x) = x^3 - 3x^2$, decide whether f increases on $(-1, 0)$ and whether it is concave up there. Locate its inflection; justify using the graph of f' .

Height of f' answers one question; increasing or decreasing f' answers another.

Positive derivative can accompany concave down

$$f'(x) = 3x^2 - 6x, \quad f''(x) = 6x - 6.$$

On $(-1, 0)$

$f' > 0$, so f increases. But f' decreases, so f is **concave down**. These conclusions are compatible.

Across $x = 1$

$f' < 0$ for $x < 1$ and $f' > 0$ for $x > 1$. Concavity changes from down to **concave up**, giving a **point of inflection** at $(1, -2)$ for the stated cubic. Here $f'(1) = -3$, not zero.

Does a zero second derivative prove inflection?

$$h(x) = x^4, \quad k(x) = x^3.$$

Your turn, at $x = 0$

Both second derivatives are zero. Find their second derivatives and test the signs on both sides. Which graph has an inflection point? Does either function have a local minimum?

A candidate needs a sign or concavity check.

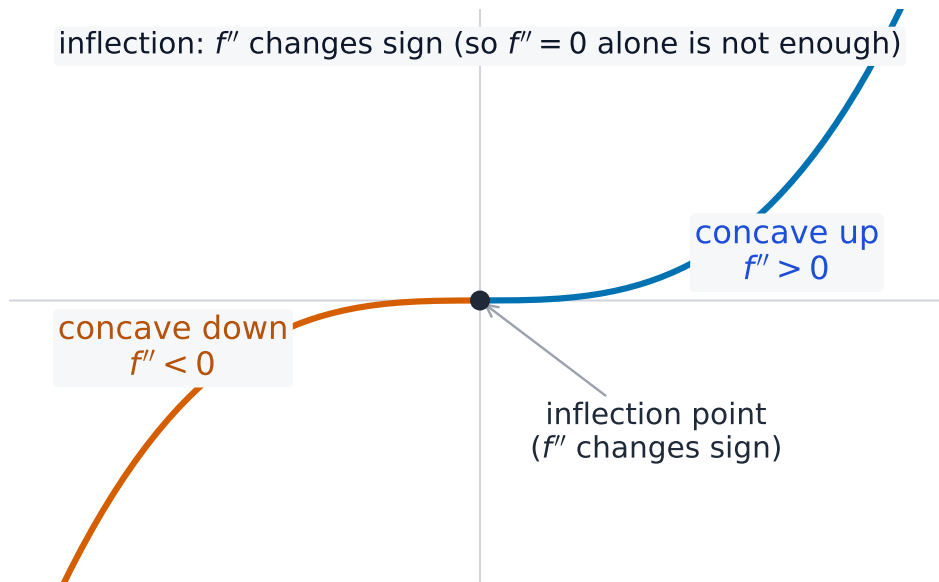
The same candidate gives different conclusions

	$h(x) = x^4$	$k(x) = x^3$
Second derivative	$12x^2$	$6x$
Signs across 0	$+$ $\rightarrow$ $+$	$-$ $\rightarrow$ $+$
Inflection at 0?	no	yes

A second warning about derivative tests

$h(x) \geq 0 = h(0)$, so h has a local minimum at 0 despite $h''(0) = 0$. A positive second derivative is a sufficient test for a minimum when the first derivative is zero; it is not a necessary condition.

Concavity and points of inflection, in detail



Separate example $f(x) = x^3/6$: concavity changes at 0.

Exercises 2.1-2.3

2.1 State where f is concave up.

[1]

2.2 For $f(x) = x^3$, find f'' and where it is concave up.

[2]

2.3 State what an inflection point is.

[1]

Exercises 2.1·2.3

2.1 State where f is concave up. [1]

2.1

- where $f'' > 0$
-

2.2 For $f(x) = x^3$, find f'' and where it is concave up. [2]

2.2

- $f''(x) = 6x$; concave up for $x > 0$
-

2.3 State what an inflection point is. [1]

2.3

- a point where concavity changes (from up to down or vice versa)

Exercises 3.1-3.2

3.1 f is concave up where

[1]

☐

A $f' > 0$

B $f'' > 0$

C $f'' < 0$

D $f = 0$

☐

3.2 An inflection point is where

[1]

A $f' = 0$

B concavity changes

C $f = 0$

D f'' is constant

Exercises 3.1-3.2

3.1 f is concave up where

[1]

☐

A $f' > 0$

B $f'' > 0$

C $f'' < 0$

D $f = 0$

3.1 B

- $f'' > 0$.

3.2 An inflection point is where

[1]

☐

A $f' = 0$

B concavity changes

C $f = 0$

D f'' is constant

3.2 B

- concavity changes.

Exercise 3.3 (a)

3.3 $f(x) = x^3$.

(a) Find $f''(x)$.

[1]

Exercise 3.3 (a)

3.3 $f(x) = x^3$.

(a) Find $f''(x)$.

[1]

(a)

- $6x$

Exercise 3.3 (b)

3.3 $f(x) = x^3$.

(b) State where $f'' > 0$.

[1]

Exercise 3.3 (b)

3.3 $f(x) = x^3$.

(b) State where $f'' > 0$.

[1]

(b)

- $x > 0$

Exercise 3.3 (c)

3.3 $f(x) = x^3$.

(c) State the inflection point.

[1]

Exercise 3.3 (c)

3.3 $f(x) = x^3$.

(c) State the inflection point.

[1]

(c)

- $x = 0$