

Exercise walk-through

5.11

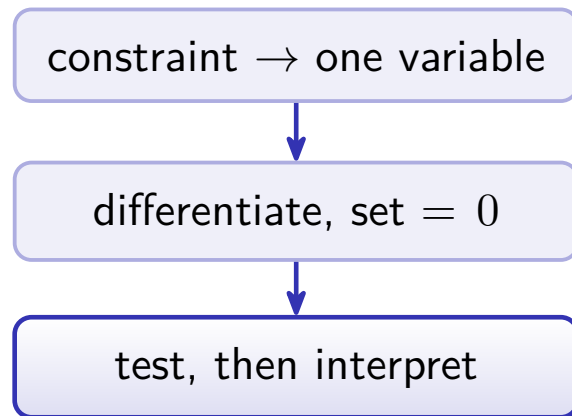
Solving Optimization Problems

AP Calculus AB

Solving an optimization problem

Optimization is the same machinery aimed at a real goal:

- 1 write the quantity as a function of **one** variable (use the constraint to eliminate the rest)
- 2 state the allowed interval
- 3 find and **test** the critical points
- 4 answer with **units** and interpretation



Worked example – the farmer's fence

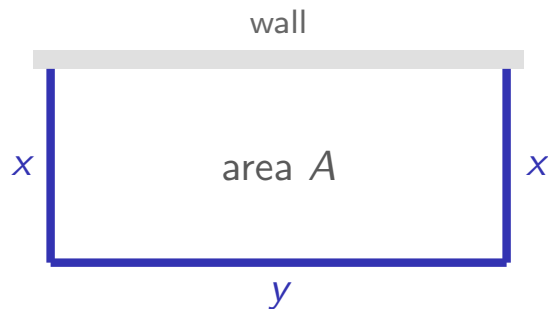
100 m of fence, three sides (a wall is the fourth). Maximize the area.

Ends x , side y : constraint $2x + y = 100$, so $y = 100 - 2x$.

$$A(x) = x(100 - 2x) = 100x - 2x^2$$

$$A'(x) = 100 - 4x = 0 \Rightarrow x = 25$$

$A'' = -4 < 0$, a maximum: $y = 50$ and $A = 1250 \text{ m}^2$.



Behaviour of implicit relations

It all extends to **implicitly** defined relations:

- horizontal tangent where $\frac{dy}{dx} = 0$
- vertical tangent where $\frac{dy}{dx}$ is **undefined**

$\frac{dy}{dx}$ usually involves **both** x and y – substitute it back in when finding $\frac{d^2y}{dx^2}$, then read concavity from the sign.

Exercises 2.1·2.3

2.1 For $A(y) = 40y - 2y^2$, find $A'(y)$ and solve $A'(y) = 0$. [2]

2.2 The area function is $A = 12x - x^2$. Find the width x that maximizes the area, and **justify** it is a maximum. [3]

2.3 A function $C(x) = x^2 + \frac{16}{x}$ models a cost for $x > 0$. Find the x that minimizes C . [3]

Exercises 2.1-2.3

2.1 For $A(y) = 40y - 2y^2$, find $A'(y)$ and solve $A'(y) = 0$. [2]

2.1

- $A'(y) = 40 - 4y$; $40 - 4y = 0 \Rightarrow y = 10$
-

2.2 The area function is $A = 12x - x^2$. Find the width x that maximizes the area, and **justify** it is a maximum. [3]

2.2

- $A'(x) = 12 - 2x = 0 \Rightarrow x = 6$; $A''(x) = -2 < 0$, so $x = 6$ is a maximum
-

2.3 A function $C(x) = x^2 + \frac{16}{x}$ models a cost for $x > 0$. Find the x that minimizes C . [3]

2.3

- $C'(x) = 2x - \frac{16}{x^2} = 0$; $2x^3 = 16 \Rightarrow x^3 = 8 \Rightarrow x = 2$

Exercise 3.1

3.1 A critical point of f has $f'(c) = 0$ and $f''(c) < 0$. The point is a [1]

☐

A local minimum

B local maximum

C point of inflection

D endpoint

Exercise 3.1

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D endpoint

3.1 B

- f'

(c)

- $= 0$ with f''

(c)

- < 0 is a local maximum

Exercise 3.2 (a)

3.2 A rectangular field of area $A = x(200 - x)$ m² is enclosed, where $0 < x < 200$.

(a) Find the value of x that maximizes A .

[2]

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(a) Find the value of x that maximizes A .

[2]

(a)

- $A = 200x - x^2, A'(x) = 200 - 2x = 0 \Rightarrow x = 100$

Exercise 3.2 (b)

3.2 A rectangular field of area $A = x(200 - x)$ m² is enclosed, where $0 < x < 200$.

(b) Justify that your value gives a maximum.

[1]

Exercise 3.2 (b)

3.2 A rectangular field of area $A = x(200 - x)$ m² is enclosed, where $0 < x < 200$.

(b) Justify that your value gives a maximum.

[1]

(b)

- $A''(x) = -2 < 0$, so it is a maximum

Exercise 3.2 (c)

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(c) State the maximum area.

[1]

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3.2 A rectangular field of area $A = x(200 - x)$ m² is enclosed, where $0 < x < 200$.

(c) State the maximum area.

[1]

(c)

- $A = 100(100) = 10\,000 \text{ m}^2$

Exercise 3.3 (a)

3.3 An open box is made from card, with volume $V(x) = 150x - \frac{1}{4}x^3$ for $0 < x < \sqrt{600}$.

(a) Show that $V'(x) = 150 - \frac{3}{4}x^2$. [1]

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(a)

- $V'(x) = 150 - \frac{3}{4}x^2$

Exercise 3.3 (b)

3.3 An open box is made from card, with volume $V(x) = 150x - \frac{1}{4}x^3$ for $0 < x < \sqrt{600}$.

(b) Find the value of x that maximizes the volume, and justify it. [3]

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3.3 An open box is made from card, with volume $V(x) = 150x - \frac{1}{4}x^3$ for $0 < x < \sqrt{600}$.

(b) Find the value of x that maximizes the volume, and justify it. [3]

(b)

- $150 - \frac{3}{4}x^2 = 0 \Rightarrow x^2 = 200 \Rightarrow x = \sqrt{200} \approx 14.1$; $V''(x) = -\frac{3}{2}x < 0$ for $x > 0$, so it is a maximum