

Exercise walk-through

4.5

Solving Related Rates Problems

AP Calculus AB

Related rates – the five-step template

In a **related rates** problem, quantities change together in time. The engine is the **chain rule**: differentiate with respect to t .

- 1 name the variables; list given & wanted rates
- 2 write an equation relating them
- 3 differentiate both sides in t
- 4 substitute the values **last**
- 5 state **units** and the correct **sign**

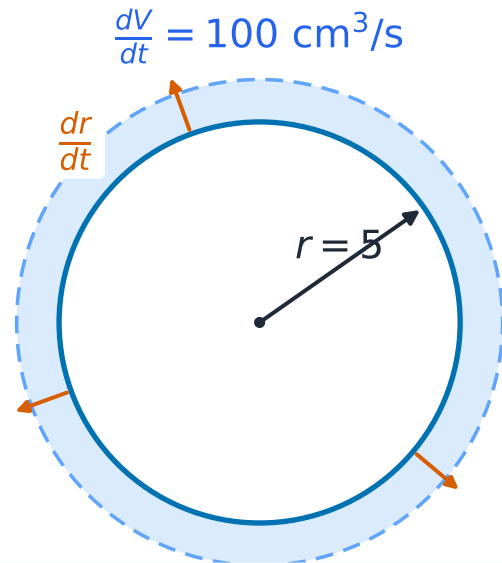
Worked example – the inflating balloon

$\frac{dV}{dt} = 100 \text{ cm}^3/\text{s}$: how fast is r growing at $r = 5$?

Start from $V = \frac{4}{3}\pi r^3$ and differentiate in t first:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$100 = 4\pi(5)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{\pi} \approx 0.32 \text{ cm/s.}$$



$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \quad (\text{chain rule on } V = \frac{4}{3}\pi r^3)$$

An inflating balloon links dV/dt and dr/dt through the chain rule

The classic related-rates error

 **Substitute $r = 5$ first**

$V = \frac{4}{3}\pi(5)^3$ is a **constant**, so
 $\frac{dV}{dt} = 0$. The radius is now
frozen – the problem is destroyed.

 **Differentiate first**

Keep r a **function of t** , differentiate the general relationship, *then* put $r = 5$ in.

Substituting the instant's value **too early** is the single most common lost mark here.

Exercises 2.1-2.3

2.1 List the steps of a related-rates solution.

[1]

2.2 A circle's r grows at 2 cm/s. At $r = 3$, find $\frac{dA}{dt}$ (with $A = \pi r^2$).

[2]

2.3 State the units of $\frac{dA}{dt}$ here.

[1]

Exercises 2.1-2.3

2.1 List the steps of a related-rates solution. [1]

2.1

- write the relation; differentiate with respect to t ; substitute; solve for the unknown rate

2.2 A circle's r grows at 2 cm/s. At $r = 3$, find $\frac{dA}{dt}$ (with $A = \pi r^2$). [2]

2.2

- $\frac{dA}{dt} = 2\pi(3)(2) = 12\pi \text{ cm}^2/\text{s}$

2.3 State the units of $\frac{dA}{dt}$ here. [1]

2.3

- cm^2/s

Exercises 3.1-3.2

3.1 After differentiating, the next step is to

[1]

☐

A stop

B substitute known values and solve

C integrate

D graph

3.2 If $\frac{dr}{dt} = 2$ and $r = 3$, then $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ equals

[1]

☐

A 6π

B 12π

C 4π

D 9π

Exercises 3.1-3.2

3.1 After differentiating, the next step is to

[1]

☐

A stop

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C integrate

D graph

3.1 B

- substitute known values and solve.

3.2 If $\frac{dr}{dt} = 2$ and $r = 3$, then $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ equals

[1]

☐

A 6π

B 12π

C 4π

D 9π

3.2 B

- 12π .

Exercise 3.3 (a)

3.3 $A = \pi r^2$, $\frac{dr}{dt} = 2$, $r = 3$.

(a) Write $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$.

[1]

Exercise 3.3 (a)

3.3 $A = \pi r^2$, $\frac{dr}{dt} = 2$, $r = 3$.

(a) Write $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$.

[1]

(a)

- $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

Exercise 3.3 (b)

3.3 $A = \pi r^2$, $\frac{dr}{dt} = 2$, $r = 3$.

(b) Substitute.

[1]

Exercise 3.3 (b)

3.3 $A = \pi r^2, \frac{dr}{dt} = 2, r = 3.$

(b) Substitute.

[1]

(b)

- $2\pi(3)(2)$

Exercise 3.3 (c)

3.3 $A = \pi r^2$, $\frac{dr}{dt} = 2$, $r = 3$.

(c) State $\frac{dA}{dt}$.

[1]

Exercise 3.3 (c)

3.3 $A = \pi r^2$, $\frac{dr}{dt} = 2$, $r = 3$.

(c) State $\frac{dA}{dt}$.

[1]

(c)

- $12\pi \text{ cm}^2/\text{s}$