

Exercise walk-through

4.4

Introduction to Related Rates

AP Calculus AB

Related rates – the five-step template

In a **related rates** problem, quantities change together in time. The engine is the **chain rule**: differentiate with respect to t .

- 1 name the variables; list given & wanted rates
- 2 write an equation relating them
- 3 differentiate both sides in t
- 4 substitute the values **last**
- 5 state **units** and the correct **sign**

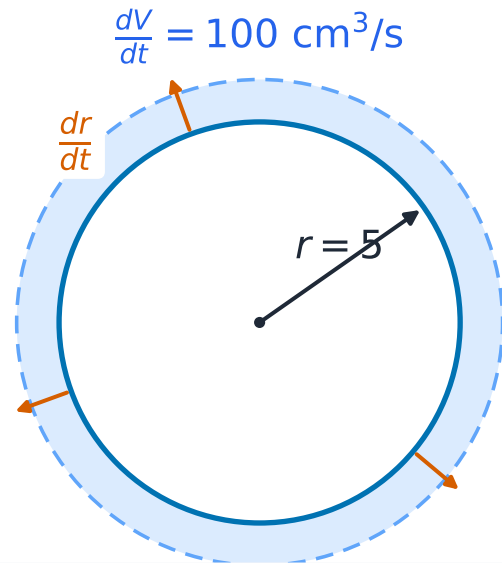
Worked example – the inflating balloon

$\frac{dV}{dt} = 100 \text{ cm}^3/\text{s}$: how fast is r growing at $r = 5$?

Start from $V = \frac{4}{3}\pi r^3$ and differentiate in t first:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$100 = 4\pi(5)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{\pi} \approx 0.32 \text{ cm/s.}$$



$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \quad (\text{chain rule on } V = \frac{4}{3}\pi r^3)$$

An inflating balloon links dV/dt and dr/dt through the chain rule

The classic related-rates error

✗ Substitute $r = 5$ first

$V = \frac{4}{3}\pi(5)^3$ is a **constant**, so
 $\frac{dV}{dt} = 0$. The radius is now
frozen – the problem is destroyed.

✓ Differentiate first

Keep r a **function of t** , differentiate the general relationship, *then* put $r = 5$ in.

Substituting the instant's value **too early** is the single most common lost mark here.

Exercises 2.1-2.3

2.1 State the first step in a related-rates problem.

[1]

2.2 Differentiate $A = \pi r^2$ with respect to t .

[2]

2.3 State what links the two rates.

[1]

Exercises 2.1-2.3

2.1 State the first step in a related-rates problem. [1]

2.1

- write the relation connecting the quantities
-

2.2 Differentiate $A = \pi r^2$ with respect to t . [2]

2.2

- $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$
-

2.3 State what links the two rates. [1]

2.3

- the derivative of the relation with respect to t

Exercises 3.1·3.2

3.1 In related rates, you differentiate the relation with respect to

[1]

☐

A x

B time t

C r

D nothing

3.2 Differentiating $A = \pi r^2$ with respect to t gives

[1]

☐

A $2\pi r$

B $2\pi r \frac{dr}{dt}$

C πr^2

D $2r$

Exercises 3.1-3.2

3.1 In related rates, you differentiate the relation with respect to

[1]

☐

A x

B time t

C r

D nothing

3.1 B

- time t .

3.2 Differentiating $A = \pi r^2$ with respect to t gives

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A $2\pi r$

B $2\pi r \frac{dr}{dt}$

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3.2 B

- $2\pi r \frac{dr}{dt}$.

Exercise 3.3 (a)

3.3 $V = s^3$ (a cube).

(a) Differentiate with respect to t .

[1]

Exercise 3.3 (a)

3.3 $V = s^3$ (a cube).

(a) Differentiate with respect to t .

[1]

(a)

- $\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

Exercise 3.3 (b)

3.3 $V = s^3$ (a cube).

(b) State the chain-rule factor.

[1]

Exercise 3.3 (b)

3.3 $V = s^3$ (a cube).

(b) State the chain-rule factor.

[1]

(b)

- $\frac{ds}{dt}$

Exercise 3.3 (c)

3.3 $V = s^3$ (a cube).

(c) Write $\frac{dV}{dt}$.

[1]

Exercise 3.3 (c)

3.3 $V = s^3$ (a cube).

(c) Write $\frac{dV}{dt}$.

[1]

(c)

- $\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$