Exercise walk-through

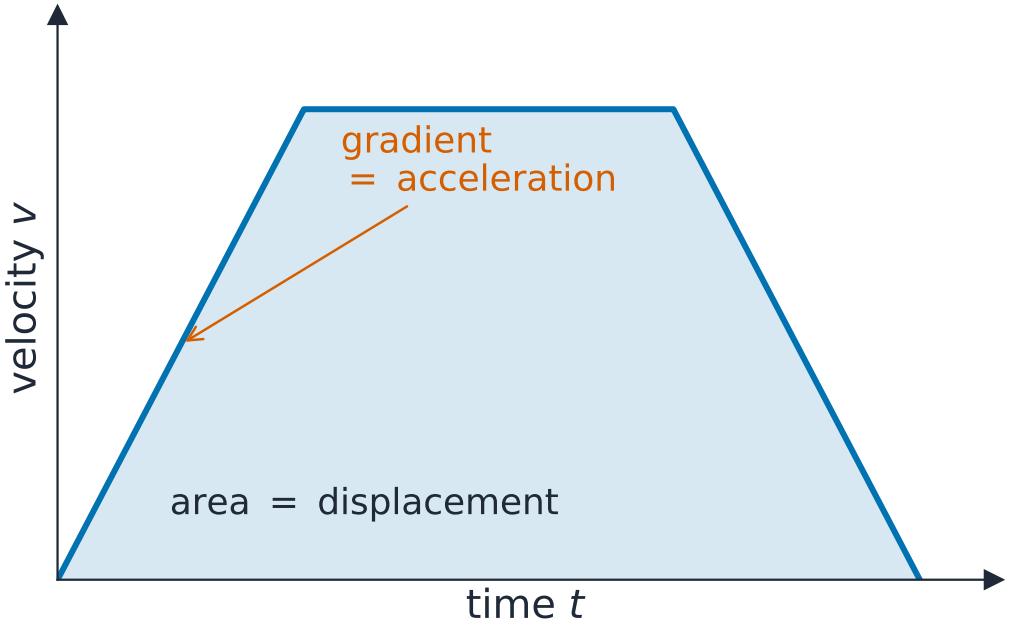
4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration

AP Calculus AB

Straight-line motion

Three functions of time, linked by differentiation:

- **position** $s(t)$
- **velocity** $v(t) = s'(t)$ – signed
- **acceleration** $a(t) = v'(t) = s''(t)$



Reading the motion – the frequent exam parts

At rest

$$v(t) = 0$$

Direction

right/up if $v > 0$, left/down if $v < 0$

Changes direction

where v changes sign

Speed

$|v(t)|$ – no direction

A rising balloon: related rates link how fast radius, volume and height change together

Speeding up or slowing down?

Compare the signs of v and a:

> Same sign $\Rightarrow$ speeding up. Opposite signs $\Rightarrow$ slowing down.

Velocity has direction; speed does not. The exam tests exactly this difference.

Roller coaster

Worked example – a particle on a line

$s(t) = t^3 - 6t^2 + 9t$

$$v(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$$

At rest at $t = 1$ and $t = 3$; it changes direction at each. At $t = 2$: $v(2) = -3 < 0$ (moving left), and just after, $a > 0$ while $v < 0$ – opposite signs, so it is slowing down.

$$s(t) = t^3 - 6t^2 + 9t$$

↓

$$v(t) = 3(t-1)(t-3)$$

↓

$$a(t) = 6t - 12$$

Exercises 2.1·2.3

2.1 State velocity in terms of position. [1]

2.2 State acceleration in terms of velocity. [1]

2.3 For $x(t) = t^2 - 4t$, find $v(t)$ and the time at rest. [2]

Exercises 2.1·2.3

2.1 State velocity in terms of position. [1]

2.1

- $v(t) = x'(t)$

2.2 State acceleration in terms of velocity. [1]

2.2

- $a(t) = v'(t)$

2.3 For $x(t) = t^2 - 4t$, find $v(t)$ and the time at rest. [2]

2.3

- $v(t) = 2t - 4$; at rest when $2t - 4 = 0$, i.e. $t = 2$

Exercises 3.1·3.2

3.1 Velocity is the derivative of [1]

- **A** acceleration
- **B** position
- **C** speed
- **D** distance

3.2 The object is at rest when [1]

- **A** $a = 0$
- **B** $v = 0$
- **C** $x = 0$
- **D** $t = 0$

Exercises 3.1·3.2

3.1 Velocity is the derivative of [1]

- **A** acceleration
- **B** position
- **C** speed
- **D** distance

3.1 B

- position.

3.2 The object is at rest when [1]

- **A** $a = 0$
- **B** $v = 0$
- **C** $x = 0$
- **D** $t = 0$

3.2 B

- $v = 0$.

Exercise 3.3 (a)

3.3 $x(t) = t^2 - 4t$.

(a) Find $v(t)$. [1]

Exercise 3.3 (a)

3.3 $x(t) = t^2 - 4t$.

(a) Find $v(t)$. [1]

(a)

- $v(t) = 2t - 4$

Exercise 3.3 (b)

3.3 $x(t) = t^2 - 4t$.

(b) Find $a(t)$. [1]

Exercise 3.3 (b)

3.3 $x(t) = t^2 - 4t$.

(b) Find $a(t)$. [1]

(b)

- $a(t) = 2$

Exercise 3.3 (c)

3.3 $x(t) = t^2 - 4t$.

(c) Find when $v = 0$. [1]

Exercise 3.3 (c)

3.3 $x(t) = t^2 - 4t$.

(c) Find when $v = 0$. [1]

(c)

- $t = 2$