

Exercise walk-through

3.4

Differentiating Inverse Trigonometric Functions

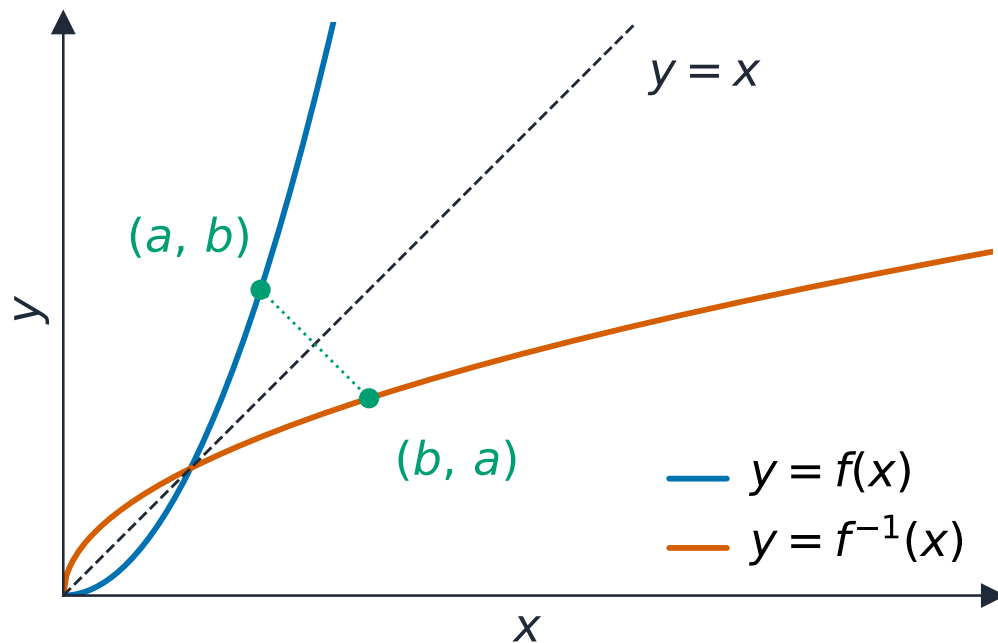
AP Calculus AB

Derivatives of inverse functions

If g is the **inverse function** of f (so $f(g(x)) = x$):

$$g'(x) = \frac{1}{f'(g(x))}.$$

Slopes at **matching** points are **reciprocals** – the graphs are mirror images in $y = x$.



Worked example – inverse from a point

$f(2) = 5$ and $f'(2) = 3$; g is the inverse of f

$(2, 5)$ lies on f , so $(5, 2)$ lies on g . Then

$$g'(5) = \frac{1}{f'(2)} = \frac{1}{3}.$$

Swap the coordinates **first**, then take the reciprocal of f' at the **original** x .

on f : $(2, 5)$

↓ swap

on g : $(5, 2)$

Inverse trigonometric derivatives

The same idea gives the **inverse trigonometric functions** you must know:

$$\begin{aligned}\frac{d}{dx} \arcsin x \\ = \frac{1}{\sqrt{1-x^2}}\end{aligned}$$

$$\begin{aligned}\frac{d}{dx} \arctan x \\ = \frac{1}{1+x^2}\end{aligned}$$

$$\begin{aligned}\frac{d}{dx} \operatorname{arcsec} x \\ = \frac{1}{|x|\sqrt{x^2-1}}\end{aligned}$$

Chain them when the input is a function: $\frac{d}{dx} \arctan(3x) = \frac{3}{1+9x^2}$.

Exercises 2.1-2.3

2.1 State $\frac{d}{dx}[\arcsin x]$.

[1]

2.2 State $\frac{d}{dx}[\arctan x]$.

[1]

2.3 State $\frac{d}{dx}[\arccos x]$.

[2]

Exercises 2.1-2.3

2.1 State $\frac{d}{dx}[\arcsin x]$.

[1]

2.1

- $\frac{1}{\sqrt{1-x^2}}$

2.2 State $\frac{d}{dx}[\arctan x]$.

[1]

2.2

- $\frac{1}{1+x^2}$

2.3 State $\frac{d}{dx}[\arccos x]$.

[2]

2.3

- $-\frac{1}{\sqrt{1-x^2}}$

Exercise 3.1

3.1 $\frac{d}{dx}[\arctan x]$ equals

[1]

☐

A $\frac{1}{\sqrt{1-x^2}}$

B $\frac{1}{1+x^2}$

C $-\frac{1}{1+x^2}$

D $\frac{1}{x}$

Exercise 3.1

3.1 $\frac{d}{dx}[\arctan x]$ equals

[1]

- ☐ A $\frac{1}{\sqrt{1-x^2}}$
- ☐ B $\frac{1}{1+x^2}$
- ☐ C $-\frac{1}{1+x^2}$
- ☐ D $\frac{1}{x}$

3.1 B

• $\frac{1}{1+x^2}.$

Exercise 3.2

3.2 $\frac{d}{dx}[\arcsin x]$ equals

[1]

☐

A $\frac{1}{1+x^2}$

B $\frac{1}{\sqrt{1-x^2}}$

C $-\frac{1}{\sqrt{1-x^2}}$

D $\arccos x$

Exercise 3.2

3.2 $\frac{d}{dx}[\arcsin x]$ equals

[1]

- ☐ **A** $\frac{1}{1+x^2}$
- B** $\frac{1}{\sqrt{1-x^2}}$
- C** $-\frac{1}{\sqrt{1-x^2}}$
- D** $\arccos x$

3.2 B

• $\frac{1}{\sqrt{1-x^2}}.$

Exercise 3.3 (a)

3.3 Differentiate each.

(a) $\arcsin x$.

[1]

Exercise 3.3 (a)

3.3 Differentiate each.

(a) $\arcsin x$.

[1]

(a)

- $\frac{1}{\sqrt{1-x^2}}$

Exercise 3.3 (b)

3.3 Differentiate each.

(b) $\arctan x$.

[1]

Exercise 3.3 (b)

3.3 Differentiate each.

(b) $\arctan x$.

[1]

(b)

- $\frac{1}{1+x^2}$

Exercise 3.3 (c)

3.3 Differentiate each.

(c) $\arccos x$.

[1]

Exercise 3.3 (c)

3.3 Differentiate each.

(c) $\arccos x$.

[1]

(c)

- $-\frac{1}{\sqrt{1-x^2}}$